Sampling-Based Capacity Estimation for Unmanned Traffic Management

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Abstract—The plenary talk at DASC 2016 by Dr. Parimal Kopardekar, the Principal Investigator of NASA UTM program, highlighted understanding the role of volume, noise and spectrum considerations in airspace demand-capacity modeling as the three requests from UTM developers to the avionics research community [1]. This paper proposes initial answers to all three requests, for the case of unmanned aerial vehicles (UAVs) operating in low-altitude uncontrolled airspace above populated areas: we estimate airspace capacity under several metrics centered on traffic volume manageability, drones noise pollution and spectrum demand. Our work aids in bridging regulators and the industry, by providing policy makers with decision support tools which help to quantify technological requirements which the manufacturers must follow in order to ensure seamless operation of small unmanned aerial systems (sUAS) in an urban airspace.

I. INTRODUCTION AND RELATED WORK

Increasing number of unmanned aerial vehicles (UAVs) poses the challenge of establishing the unmanned traffic management (UTM) system. A fundamental task in UTM is capacity estimation: how much traffic can be safely accommodated and successfully managed within the given airspace? The question can be studied from various perspectives, taking into account multiple capacity-limiting factors: appearance of hard-to-resolve conflicts (if their probability is high, capacity management measures should be established), excessive noise (improved technologies may be mandated to nurture social acceptability and positive public perception of the drones industry), communication spectrum jamming (including cybersecurity considerations, as stronger encryption protocols require more bandwidth), etc.

Airspace capacity estimation is inherited by UTM from the ATM domain, where it is has been a topic of recurring research interest [2]–[8]. In contrast to ATM, which mostly deals with airport-to-airport flights scheduled and planned in advance (often even adhering to a regular pattern), the UTM is facing a larger number of vehicles and users with less predictable demands, diverse flying experience and the ability to start/end the trips essentially anywhere. That is, one important difference between UTM and ATM is the much stronger *non-deterministic* component inherent to the small unmanned aerial systems (sUAS) traffic. Vishwanath Bulusu Cal Unmanned Aviation Lab Civil and Environmental Engineering Department University of California Berkeley, California, USA vishwab@berkeley.edu

In this paper we use the probabilistic map of UAV traffic to give sampling-based capacity estimations. We call our setup the *Likely UTM* (or *LiU*) model. To put our sampling approach into perspective of the large variety of stochastic models for air traffic, used previously in the literature, we mention here two settings on which our LiU model builds up:

- The probabilistic setup which [9] called the *Dutch model*, was used in PhD thesis of Hoekstra [10], developed by Jardin [11], and more recently explored within the Metropolis project by TU Delft [12]. In this model the aircraft are distributed uniformly in the given airspace. In the basic version of the Dutch model, the direction of flight is also uniformly distributed in 0...360, while in the most recent work [8] different direction cones are separated by altitude. The simplicity of the model allows one to obtain exact formulas for conflict probabilities and other quantities, using a single parameter tuned to match empirical data (in [10, p. 220] the parameter is the probability p_{c_a} of conflict in the air, estimated using real traffic observation; in [11] the parameter p_t is the ratio of the so called "element" to the area swept by an aircraft during the observation period).
- An extension of the Dutch model is *Cal model*¹. In Cal model, flights endpoints are sampled based on the population density (and hence neither the vehicles locations nor their headings are distributed uniformly), and each flight is a straightline segment connecting the endpoints elevated to a given height *h* (Fig. 1). As in the (basic version of) Dutch model, all drones fly with the same speed and occupy a single level (h = 50m), so the setup is essentially two-dimensional. Existing results for Cal model were obtained using simulation of the traffic.

Our LiU model is based on the simple idea, in a sense combining Dutch model in which the probability of observing an aircraft in any region is known (the probability is uniform) but the origin-destination of the flights are not specified, and

¹The name "Cal" was chosen in [13] for two reasons: from the fact that the model was introduced by researchers representing University of California Berkeley (going by Cal) [14], and from the vision expressed by Dr. Kopardekar (representing California-based NASA Research Center) [15]: "every home will have a drone and every home will serve as an aerodrome".



Fig. 1. Fig. 3 from [14]: A typical UAS flight path in the Cal model

Cal model in which the origin-destinations are generated but the probability of seeing a vehicle in any region is not known. LiU model works with the pointwise distribution of the traffic (similarly to the Dutch model), but the probability is not uniform (similarly to the Cal model): using Cal model's origindestination spawning and direct routing, we *precompute* the probability of having a vehicle at any point in the domain (unlike in the Dutch model, the probability is not uniform) and use the probabilities to *generate* a snapshot of the traffic (differently from existing work on Cal model, where snapshots were simulated). The snapshots are processed to estimate the measures of interest; we also employ the probability distribution directly to compute expectations of the measures.

As applications of LiU model, we reproduce, nuance and extend some of the existing simulation results, as well as provide several new estimations both on old problems and on a new frontier. The experiments were run for two metropolitan regions: Bay Area in the US and Norrköping municipality in Sweden.

Among our results are the following:

- We estimate airspace capacity defined as the number of drones at which safety becomes compromised due to frequent conflicts involving more than k=3 vehicles (the definition is taken from [16], where the same capacity estimations were obtained via simulations). LiU model allows us to compute the capacity faster; in addition, we estimate the capacity for a wider range of the critical deconfliction number k (for k=2 we show how to calculate *exact* expected values).
- We compute ambient noise levels that may be generated by future drones operations, reproducing simulated metrics from [13]. Again, our computations are more precise and time-efficient than the simulations; we also simulate an additional metric – so called N55 contours [17].
- We give estimates of spectrum demand, for various operational modes of sUAS.

	Simulation	Sampling	Calculation
Conflicts	[16]	Sec III	Sec III (collisions)
Noise	[13] (L_{eq}, L_{10}) , [17] (N55), Sec IV (N55)	Sec IV-A (L_{10})	Sec IV (L_{eq})
Spectrum	[18]		Sec V
TABLE I			

MEASURES (ROWS) OBTAINED WITH DIFFERENT METHODS (COLUMNS). CONTRIBUTIONS OF THIS PAPER REFER TO SECTIONS WHERE THE MEASURES ARE ESTIMATED

Table I summarizes existing and new results.

The rest of the paper is organized as follows: The next section describes how spatial distribution of UAVs is obtained. Section III is devoted to conflict rate investigations, using random geometric graphs. In Section IV we use our model to assess ambient noise levels generated by sUAS. In particular, in both sections we report on a series of computational experiments demonstrating efficiency of our sampling-based algorithms: in all experiments, our implementations outperform the earlier simulation-based methods, allowing us to reproduce some existing results faster and more accurately than in the prior work, as well as to obtain new results. Section V presents estimates of sUAS spectrum demand. Finally, Section VI concludes the paper and discusses possible extensions.

II. THE DRONE MAP

LiU model has the same inputs as the earlier, simulationbased methods for capacity estimation:

- the region of interest R
- population density D(g) specified for every point $g \in R$
- the duration T of the period of interest (typically T = 12hrs [9], [13], [14], [16], i.e., the traffic was simulated over a day)
- the expected number N of UAV operations during T (the parameter N is varied during the experiments to understand the effect of traffic density on the system)

We focus on very low level (VLL) uncontrolled airspace over populated areas and assume that the demand for the airspace is generated according to Cal model (arguments in favor of our choices may be found in [13], [19], [20]; alternatively, any other demand pattern may be used – e.g., UAXPAN UAV operations forecasts from Mosaic [21], [22]): the start times of the flights from any point $a \in R$ form Poisson process whose intensity $\lambda_s(a) = \frac{N}{T} \frac{D(a)}{\int_R D \, dA}$ is proportional to the population density at the point, and the destination *b* of a flight is chosen at random based on the density as well – the probability that the flight ends at a point $b \in R$ is $p(b) = D(b) / \int_B D \, dA$.

The core of LiU model is computing the pointwise distribution of the traffic. The crucial observation is that for any point $g \in ab$, along the segment between pixels a and b, the drones flying from a to b appear over g according to Poisson process with intensity $\lambda_{ab} = \lambda_s(a)p(b) = \frac{N}{T} \frac{D(a)D(b)}{(\int_R D \, dA)^2}$ – this is a direct consequence of the fact that the Poisson process of the trip origins decomposes based on the destinations. Integrating over all origin-destination pairs, we obtain that overall the



Fig. 2. $\lambda(g) = \int_{ab \ni g} \lambda_{ab} \, dA \, dB$ (left), which is approximated by $\lambda(g) = \lambda_{ab} + \lambda_{a'b'} + \lambda_{a''b''} + \dots$ in the discrete case (right)

drones appear over any point g according to the compound Poisson process with intensity $\lambda(g) = \int_{ab \ni g} \lambda_{ab} \, dA \, dB = \frac{1}{(\int_R D \, dA)^2} \frac{N}{T} \int_{ab \ni g} D(a)D(b) \, dA \, dB$ where $ab \ni g = \{(a, b) \in R^2 : g \in ab\}$ is the set of endpoints of all segments containing g (Fig. 2, left).

Of course, in practice, everything is discretized: a grid L is laid over R and the population density D(g) is given for each grid pixel $g \in L$ (see Fig. 9). The start times of the flights from any grid pixel $a \in L$ form Poisson process whose intensity $\lambda_s(a) = \frac{N}{T} \frac{D(a)}{\sum_{x \in L} D(x)}$ is proportional to the population density at the pixel, and the destination pixel *b* of a flight is chosen at random based on the density as well - the probability that the flight ends at a pixel $b \in L$ is $p(b) = D(b) / \sum_{x \in L} D(x)$. The straightline path of UAV between pixels a and b is represented by a sequence of grid pixels through which the drone goes (we use Bresenham's line drawing [23] to snap straightline segments onto the grid); we identify the sequence with the segment ab. We assume that the drone spends the same time t = l/v, where l is the pixel side length and v is the drones speed; similarly to the earlier papers [9], [14], [16], we work with l = 150m and v = 25m/s, leading to t = 6s. With a fine grid, the discretization serves as a reasonable approximation to the continuous setting.

Similarly to the continuous case, for any pixel $g \in ab$ the drones flying from a to b enter g according to Poisson process with intensity $\lambda_{ab} = \lambda_s(a)p(b) = \frac{N}{T} \frac{D(a)D(b)}{\sum_{x \in L} D(x)}$ (again, this is a direct consequence of the fact that the Poisson process of the trip origins decomposes based on the destinations). Summing over all origin-destination pairs, we obtain that overall the drones enter any pixel g according to the compound Poisson process with intensity $\lambda(g) = \sum_{ab \ni g} \lambda_{ab} = \frac{1}{(\sum_{x \in L} D(x))^2} \frac{N}{T} \sum_{ab \ni g} D(a)D(b)$, where now $ab \ni g = \{(a, b) \in L^2 : g \in ab\}$ are the endpoints of the (pixelized) segments containing g (Fig. 2, right).

We call the (graph of the) function $m(g) = \sum_{ab \ge g} D(a)D(b)$ the *drone map*; the map shows how likely it is to see the drones at different pixels. Computing the map takes time cubic in the number of pixels (but computation is easily parallelizable). After the map is computed, a snapshot of the drone traffic can be obtained simply by sampling the map. Fig. 3 illustrates the map over Norrköping and a sample from it.

Overall, the number of drones n(g) that one can see in pixel g during time t is the Poisson r.v. with parameter

 $\bar{\lambda}(g) = \lambda(g)t = \frac{t}{T(\sum_{x \in L} D(x))^2} Nm(g)$. In our experiments, the density and the values *t* and *T* do not change (we vary only *N* and another parameter, *r*, introduced in the next section). Therefore, to simplify the formulas, we normalize the density so that $\frac{t}{T(\sum_{x \in L} D(x))^2} = 1$, which implies

$$\bar{\lambda}(g) = Nm(g) \tag{1}$$

III. LARGE-SIZE CONFLICTS AND COLLISIONS

Random geometric graphs (RGGs) play central role in our estimation of the number of conflicts and collisions for drone traffic; we thus begin with a short recap on RGGs. Let *S* be a set of *n* points randomly distributed within a given region; let r > 0 be a number. The RGG G(n, r) is an undirected graph that has points of *S* as vertices, and two vertices connected whenever the distance between them is at most *r*. For a natural number k > 0, let $p_k(n, r)$ denote the probability that G(n, r) has a connected component of size at least *k*. For instance:

- *p*₁(*n*, *r*) = 1 (because any vertex is a connected component of size 1)
- $p_2(n, r)$ is the probability that G(n, r) has an edge (an edge is a size-2 connected component)
- $p_n(n,r)$ is the probability that the graph is connected
- $p_k(n,r) = 0$ when k > n (because G(n,r) has only *n* vertices).

A celebrated result in RGG theory is that when the points are distributed uniformly, then for large *n* the probability $p_n(n, r)$ (i.e., the probability that G(n, r) is connected) exhibits a sharp threshold as a function of *r* [24]: the probability steeply jumps from (almost) 0 to (almost) 1 as *r* passes over the critical value (the threshold). In fact, it was shown that thresholds exist not only for connectivity, but also for all "monotone" properties, i.e., properties that continue to hold when edges are added to the graph [25]. In particular, since having a component of size at least *k* is a monotone property (adding edges does not decrease components size), under the uniform distribution $p_k(n, r)$ has a sharp threshold for any *k*.

The probabilities $p_k(n, r)$ are of interest in UTM for the following reasons. Assume that *r* represents loss of separation event, i.e., that two drones within distance *r* are in conflict (or even have collided, if *r* is small). Then for *n* drones, randomly distributed on their flight level, existence of a size-*k* connected component in G(n, r) means a conflict involving *k* vehicles (Fig. 4). While for small *k* (like *k*=2) the conflict may be resolved by simple rules-of-way (like maneuver-right), for larger *k*, size-*k* conflict may mean a safety event. This way, $p_k(n, r)$ represent probability of a safety breach.

In particular, it is of utmost interest in UTM to understand whether the probabilities $p_k(n, r)$ exhibit thresholds when viewed as functions of r. Indeed, the parameter r represents technological capabilities of the drones – communication and navigation precision, CD&R strength, etc. It would be nice to quantify, even approximately, what technology levels should



Fig. 3. Left: Drone map over Norrköping; blue is the densest hexagon (Section V). Right: Traffic sample



Fig. 4. Fig. 3 from [16] (where connected components were called "clusters"): "Abstraction of an airspace snapshot as a graph with cluster sizes indicated. (Aircraft in conflict are shown by red dots. The conflict pairs are connected by a black line. The arrows indicate the heading of the aircraft.)"

be mandated for the drones in order to maintain low probability of safety events (and more generally, how the probabilities depend on the equipage).

Unfortunately, despite the rich theory of RGG thresholds for *uniform* distribution (briefly surveyed above), to our knowledge no theoretical results are known for $p_k(n, r)$ for the case when the graph vertices are distributed non-uniformly (and the distribution of drones in the sky may be highly non-uniform – see, eg, Fig. 3). The probabilities were assessed in [16] using simulations, which confirmed existence of the thresholds (see e.g., top left graph in Fig. 5). Below we reinstate the results of the simulations in [16] (as well as present some new results) using the drone map.

A. Measure expectations via simulation, sampling and computation

We emphasize that due to the stochastic nature of UAV traffic, most measures of our interest are random variables, and we are estimating their *expected* values (more generally, one

may be interested in their distributions).² Given the complexity of the problem, it is likely impossible to obtain closed-form analytical expressions for the expected values; instead, they may be estimated via simulations, sampling or computation. The remainder of this section describes applications of the methods to RGGs; the next section discusses applications to sUAS noise impact.

Sampling vs. simulation

First, we make a little technical change: instead of looking at p_k as a function of n (the sample size), we look at it as a function of N (the overall expected daily traffic volume); this is an equivalent view, since n and N are linearly related to each other (summing equation (1) over the whole grid, it can be seen that for a fixed population density, the expected sample size is proportional to N). We will thus be concerned with estimating $p_k(N, r)$ – the probability of observing a connected component of size at least k in the snapshot of the traffic of daily intensity N.

In [16], the probabilities $p_3(N, r)$ were estimated for a set of Ns between 10 and 200000, and a set of rs between 5 and 300m. The results are shown in top left graph in Fig. 5. Computing the probabilities took 48hrs. Using LiU model, we reinstated the results from [16] in 3hrs; our graph (for an even wider ranges of N and r) is in the middle top in Fig. 5. We emphasize that the drone map is computed only *once* and works *all* N – in sharp contrast to the simulation approach where a separate run was done for each N.

We estimated the probabilities p_k also for other k. To quote [16], "Since transportation practice tends to eliminate all free de-confliction problems with more than three vehicles,

²A technical note: it is common to view the probability of an event *E* (e.g., of existence of a large connected component in the drones conflict graph) as the expected value of the indicator r.v. 1_E of the event (1_E equals 1 if *E* happens and 0 otherwise). Indeed, if *p* is the probability that *E* happens, then the expectation of the indicator r.v. is $E[1_E] = 0 \cdot (1 - p) + 1 \cdot p = p$.

we choose the acceptable size of the largest de-confliction problem as 3 and use de-confliction problem size greater than 3 as the definition of large. De-confliction literature as elaborated in [7] shows that this number might improve in future for automated real time resolutions." Our results for k=3...7 are presented in Fig. 5 (due to page limit, we sometimes show results for Bay Area and sometimes for Norrköping). As expected, for larger k, the threshold curves move into larger N and r.

Direct computation

Simulation and sampling provide approximations to the expected values – the more simulation runs or more samplings are done, the better the approximations. Having replaced simulations by sampling, we showed above how computational efficiency of the estimations may be improved; equivalently, sampling allows one to obtain better approximations using the same computational resources – CPU time, memory, etc. (precise quantification of possible quality improvements is beyond the scope of the paper). Here we describe how to calculate expected values directly from the drone map; another example is given in the next section for noise pollution.

We compute the number of conflicts, or in terms of the connected components, the expected number of components of size k=2 in the RGG on drones.³ By linearity of expectation, the total expected number of conflicts, *C*, is the sum of the expected number of conflicts over all pairs of pixels:

$$C = \sum_{g,g' \in L^2} C(g,g') + \sum_{g \in L} C(g)$$
(2)

where C(g, g') is the expected number of edges between drones in pixels g and g', and C(g) is the expected number of edges between drones in g.

Estimating inter-pixel conflicts C(g, g') may be challenging, because existence of the edge between drones $d \in g, d' \in g'$ may, in general, depend on the exact locations of the drones inside the pixels (Fig. 6, left). This may be handled differently, depending on how r and l relate to each other:

- If r ≫ l (Fig. 6, middle), the dependence of the edge existence on the exact locations may be ignored, so that C(g, g')=0 whenever g and g' are "further than r" from each other and C(g, g') = ½∑_{n,n'} nn'Pr{n(g) = n}Pr{n(g') = n'} for pixels g, g' that are "closer than r" (the division by 2 is due to each edge of RGG being counted twice). This case is not relevant for us, since even our largest r=500m is not much larger than l=150m.
- If r ~ l, one could refine the grid to have the new pixel size l' ≪ r, and apply the above. However, calculations

³We chose k=2 because it is an important measure, but also because for larger k exact computation may be too time-consuming, as one would potentially need to scroll through all k-tuples of pixels; in fact, the "intermediate" k (between small numbers and something close to n) are hard also from the theoretical perspective: while very good formulas have been derived for p_2 , p_n and p_k for large k (the emergence of the so called "giant component"), to our knowledge, for general k only rough estimates are known as closed-form solutions.

on the refined grid may take too long. We, therefore, do not follow this approach.

Finally, if $r \ll l$, inter-pixel edges may exist only between • adjacent pixels g, g'. Moreover, probability of such an edge is low (Fig. 6, right), as each drone would have to fall into width-r strip near its pixel boundary (which happens with probability $\sim (rl)/(l^2) = r/l)$, and in addition, the distance between the drones along the boundary would have to be less than r (which gives an extra probability factor of $\sim r/l$). Overall, an inter-pixel edge has probability $\sim (r/l)^3$ (e.g., for our r = 5m, l = 150m, this is about 10^{-4}) and may be ignored as a boundary effect. We thus keep only the second term (intra-pixel edges) in the formula (2) for the expected number of conflicts. Now, for n drones in a (static) snapshot, the expected number of edges in g is $\frac{1}{2}n(n-1)\frac{\pi r^2}{l^2}$ (we ignore the boundary effects once again and use $\frac{\pi r^2}{l^2}$ as the probability that a drone falls within distance r of another drone). As a drone moves during time t through the pixel, the total number of edges is

$$C(n) = \frac{1}{2}n(n-1)\frac{\pi r}{l}$$
 (3)

Recall that the number of drones in g is Poisson r.v. n(g)with parameter Nm(g) (equation (1)); thus the expected number of edges in g is $C(g) = \sum_{n} C(n) \Pr\{n(g) = n\} = \frac{\pi r}{2l} \sum_{n} n(n-1) \Pr\{n(g) = n\} = \frac{\pi r}{2l} (E[n^2(g)] - E[n(g)]) = \frac{\pi r N^2 m^2(g)}{2l}$ (since E[n(g)] = Var[n(g)] = Nm(g)). Summing this over the whole grid and over the whole day (T/t snapshots), we get the total expected daily number of conflicts

$$C_d = \frac{\pi r N^2 T}{2lt} \sum_{g \in L} m^2(g) \tag{4}$$

Remarks: (1) More precise calculations may take into account the initial inter-drone distance d_0 and relative velocity v_r : it can be shown that the drones come closer than r iff the angle α between d_0 and v_r is less than $\arcsin(r/d_0)$. One could then compute the number of edges by integrating over the distribution of d_0 and α .

(2) Interpixel edges may be accounted for with *Buffon–Laplace* probabilities [26], [27] of edges intersecting grid lines.

(3) Jardin obtained the following formula for the number of collisions [11, Eq. (10)]: $C_{\rm NR} = p_t [(D_{\rm sep} \cdot V \cdot T \cdot A)/2] \rho_{\rm AC}(\rho_{\rm AC} - 1/A)$ where $\rho_{\rm AC} = N_{\rm ss}/A$ [11, Eq. (9)], $N_{\rm ss}$ is the number of aircraft, p_t is a problem-specific parameter, $D_{\rm sep}$ is the separation distance, V is the speed, T is the observation time and A is the area. In our notation, $N_{\rm ss} \mapsto n, D_{\rm sep} \mapsto r, V \mapsto v, T \mapsto t, A \mapsto l^2$, and the formula becomes $C_{\rm NR} = \frac{1}{2} p_t rvt l^2 \frac{n}{l^2} (\frac{n}{l^2} - 1/l^2) = \frac{1}{2} n(n-1) p_t \frac{r}{l}$. Comparing this with our equation (3), we may say that in Jardin's terms, our value for p_t is π .

After the drone map is built, calculating the sum $\sum_{g \in L} m^2(g)$ is just a linear scan over the grid, which is done essentially instantaneously. Needless to say, this drastically outperforms any kind of simulations which would have to



Fig. 5. $p_k(N, r)$ for Bay Area. Top row: k = 3, 3, 4; bottom row: k = 5, 6, 7. The first graph in the top row is Fig. 7 from [16]; the other graphs are our sampling-based estimates from LiU model in this paper



Fig. 6. Existence of the edge may depend on where the drones are in the pixels (left); however, when $r \gg l$, the dependence is weak. When $r \ll l$ (right), probability of inter-pixel edge is low



Fig. 7. The expected number of conflicts (blue) and flight hours (red) as functions of ${\cal N}$

be run separately for each N. Our estimates of C_d as (the quadratic) function of N are plotted in blue in Fig. 7.

Our calculations may be tied to a recent MITRE report [28] which proposed a maximum loss of 1 flight

per 1000 flight hours over urban areas. Assume that r=5m implies collision and loss of flight. From the drone map, we calculate average duration of a flight $\tau = \frac{1}{v|L|(|L|-1)(\sum_g D(g))^2} \sum_{a,b \in L^2} |ab|D(a)D(b) \approx 2000$ sec for Bay Area (we checked the number also with simulations), where |L| is the total number of pixels. The overall expected number of flying hours $H = N\tau$ grows linearly with N, while the number of collisions C_d grows quadratically (eq. (4)). Their ratio reaches 0.001 at N = 2457 – this gives a rough traffic intensity at which UTM measures are called for.

IV. NOISE FOOTPRINT

In this section we reinstate some simulation results [13] on noise pollution from drone operations; we also obtain an estimate for another noise measure. The basic setup is the same as in [13]: any drone produces the same reference noise of L_h =60dB at the point directly under (i.e., at distance *h* from) the drone. That is, the square of the sound pressure at the point is $p_h^2 = p_0^2 \cdot 10^{L_h/10}$ where $p_0 = 20\mu$ Pa is the reference sound pressure [29, p. 240]. Spherical spreading is assumed for the sound propagation (6 dB drop in sound level per doubling of distance from the source) – see Fig. 8, where p(r) is the sound pressure at distance *r* from the point directly under the drone. The sound from different vehicles is assumed uncorrelated, and the intensities are summed up (which corresponds, e.g., to 20dB per 100-fold increase in the number of vehicles).

From the equation in Fig. 8, the sound intensity that a drone, flying in a grid pixel g, produces at a pixel ℓ is $p_{g\mapsto\ell}^2 = \frac{p_h^2 h^2}{h^2 + |g\ell|^2}$. Suppose that *n* drones flew through g during the day (i.e., during *T*=12hrs) and recall that we assume that each drone spends the same time *t* in the pixel. Then the total sound intensity from g to ℓ is $nt \frac{p_h^2 h^2}{h^2 + |g\ell|^2}$. By (1), the number of drones



Fig. 8. Fig. 4 from [13]: Pressure square follows the inverse-square law)

seen in g during t is a Poisson r.v. with parameter Nm(g); thus the expected noise intensity at ℓ from g during the whole day (i.e., T) is $\frac{T}{t}Nm(g)t\frac{p_h^2h^2}{h^2+|g\ell|^2} = TNm(g)\frac{p_h^2h^2}{h^2+|g\ell|^2}$. Summing this over all pixels, and taking the average over time (i.e., dividing by T), we obtain that the long-term average sound pressure at ℓ is $p^2(\ell) = p_h^2h^2N\sum_{g\in L}\frac{m(g)}{h^2+|g\ell|^2}$, and the long-term average expected noise level is $L_{eq}(\ell) = 10 \log_{10}(p^2(\ell)/p_0^2)$.

Computing the noise footprint according to the above formula takes 5.5 hours for Norrköping. We emphasize that the computations produce *exact* expectations for $L_{eq}(\ell)$, unlike simulations [13] or any kind of sampling, which (as usual) only approximate the expected values. Fig. 9 shows L_{eq} maps for N=5000, alongside with the approximate maps obtained via simulations in [13]. It can be seen that the simulations approximate the exactly computed contours reasonably well. However, the exact contours are much smoother; in fact, on the simulated heatmap, one can see individual flight paths taken by drones in the particular simulation instance.

Following [13], we also estimated L_{10} metric in noisy locations. The metric is the noise level that is exceeded 10% of the time. We sampled the drone map 1000 times, using N=5000 (the same as in [13]). For each sample we calculated the noise at the location of interest; L_{10} is then simply the 0.9th quantile, ie the noise level exceeded in 100 samples. The sampling and the calculations took a couple of minutes altogether (after the drone map was built) – again witnessing drastic performance improvement in comparison with hours of simulations [13].

Fig. 10 shows how L_{10} change with N; as elaborated in [13], the metric for arbitrary N is obtained by extrapolating the values for N = 5000 with a closed-form formula. As was also observed in [13], unlike the conflict probability, the noise levels do not exhibit thresholds: it may be expected that the noise will *not* jump up sharply as the number of sUAS operations increases (i.e., noise, as a capacity-limiting factor, is "better behaved" than the conflicts).

A. Audible events

We also evaluated another metric – the *number of audible* events (N55). An event is a UAV flight, and it is *audible* at a point g if it creates noise of at least 55dB at g. The metric was defined and assessed in [17] using simulations for personal unmanned vehicles in a futuristic city within Metropolis project [12]. We simulated the traffic as we did when estimating other noise metrics in [13]; the results are shown in Fig. 11. It might be interesting to compare the simulation results to some estimates from LiU model, but we did not see a good way to obtain N55 contours via sampling or exact computation.

V. SPECTRUM DEMAND

This section considers spectrum as a capacity-limiting factor. A commonly expressed view is that UTM may use existing cell towers infrastructure for communication and control (in particular, in Beyond Radio Line of Sight (BRLOS) operations mode, the towers provide a natural way for the communication signal to reach the UAV); for example, a recent SESAR study [19] suggests that "... solutions adapted to BVLOS operations occurring in VLL, especially in urban areas ... require further exploration of ... use of mobile phones network (4G, 5G etc.)", while [31] even goes as far as noting "... that future 5G standards are being specifically designed to accommodate UAS."; see also [18], [32]–[34]. Estimations of peak data rates for networks vary: earlier, [35] claimed 100Mbps downlink (DL) and 50Mbps uplink (UL) for LTE (it is also common to see 300Mbps DL and 75Mpbs UL [32], [36], [37]); currently, Verizon publicly offers more realistic 50Mbps DL [38]. Note that for video transmission from UAVs we are interested in the UL. Assuming that DL is twice faster than UL, we get 25Mbps UL for LTE. We make a conservative estimate that half of this throughput, i.e., maximum data rate of R_{max}=12.5Mbps will be available for drones (the rest going to mobile devices, etc.).

In order to give upper and lower capacity bounds based on the necessary throughput values, we look at the extremes of possible data rate requirements for sUAS:

- At the lower end, the *minimal* condition is to maintain simplest Command and Control (C2) functionality [39]. [40], i.e., use data link between the UAV and the control station for the purposes of managing the flight. In static surroundings, UAV position updates (analogous to aircraft's ADS-B/ADS-C) may be sent through a channel with low connection speed (but also with low latency, high reliability and encryption support preventing acts of unlawful interference), demanding only modest bandwidth. However, a burst of communication will happen when the environment changes (due, e.g., to weather update, appearance of a new or removal of an existing geofence, public safety UAS proximity [41, Principle 5], etc.) and the change will have to be broadcast to all flights in the cell [41, Principle 4]; in addition, the updated individual flight plans will have to be delivered simultaneously to every affected drone. Assuming that a flight plan update consists of a dozen of waypoints, each specified with a hundred of bits, we get that the cell tower would have to send the data at a rate of about a kbps/drone (this matches the throughput requirement estimations of .1-120kbps/drone from [32] and .666-5.5kbps/drone from [33]).
- The maximum bandwidth may be required for drones sending live video streams – a transmission type, characteristic to many envisioned sUAS use cases (inspection and mapping, news coverage and CSI, live map provision



Fig. 9. Noise footprint for Norrköping. Left: Simulations from [13]. Right: Calculations from LiU model in this paper. Top row: noise heatmap. Bottom row: dB contours (the pixels color-code population density (data courtesy [30]))



Fig. 10. The (logarithmic) dependence of L_{10} on traffic intensity N. Blue: Norrköping municipality. Red: Bay Area.





Fig. 11. N55 contours over Bay Area

mates in [33, Table 2], which did not assume high video quality).

We use the drone map to find the most congested tower: we consider the tiling by hexagonal cells (whose centers are spaced 2.5km apart), and find the hexagon *H* with the maximum total weight, $m(H) = \sum_{g \in H} m(g)$, of the drone map pixels inside (see Fig. 3, left) – this identifies the bottleneck area which will be most likely jammed. The number of vehicles in *H* is Poisson r.v. with parameter Nm(H). Using per-vehicle data rates from the above, we obtain that the minimum and maximum data rate requirements from the tower are Poisson r.v.'s with parameters b(N) = Nm(H)kbps and B(N) = 5Nm(H)Mbps resp.

It follows that for a "minimal" (C2-only) UTM, spectrum will probably not be an issue: even with a 3G network, supplying throughput of $R_{\text{max}}/10=1.25$ Mbps, it would take more than a thousand drones in a cell to exceed the capacity - an event of probability 10^{-16} for $N = 2 \times 10^{6}$ in a single metropolitan area, which is above even the most aggressive forecasts. We therefore concentrate on the higher end of the spectrum demand. Solid blue line in Fig. 12 shows the probability $Pr\{B(N) > R_{max}\}$ that the available bit rate will be exceeded; the probability exhibits a threshold, similarly to the probabilities p_k for RGGs (Section III). However, if the cell size is reduced by a factor of 5 (common practice in crowded areas), the threshold moves to much higher values of N (we emphasize, however, that our computations of hexagon weight m(H) and search for the heaviest hexagon are based on the *drone* map, not population density map).⁴ The figure also demonstrates effects of changing the network capacities up and down by factors of 10, representing, e.g., LTE Advanced (Release 10) and 3G networks resp. [35], [36], [38]. It can be seen that 3G network does lower the capacity significantly, as the threshold becomes less sharp and the capacity gets exceeded, with probability .1-.2 already for smaller values of N around 1000. At the same time, LTE Advanced might not need to be called for in the near future, because the threshold for LTE is already quite high (note that the N-axis on Fig. 12) is logarithmic). One may also be interested in N at which $Pr\{B(N) > R_{max}\}$ passes over e.g., 0.1 (meaning the video stream is jammed 10% of the time, or equivalently for 1/10th of the vehicles on average). With LTE, this happens already with thousands of drones per day (which may be expected in the reality), motivating employment of the Advanced networks in UTM (for which the 10% is reached well after 10000 operations).

On the methodology side, we remark that only the hexagon weight m(H) is computed from the drone map; the probability graphs are just theoretical plots of the distributions. In this sense the presented results fall into the category of exact calculations (no simulation or sampling is used to obtain them).

VI. CONCLUSION

We evaluated several measures related to UTM demand/capacity imbalance in terms of volume, noise and spectrum; on the technical side, we proposed to use sampling instead of simulation. While the sampling outperformed



Fig. 12. Solid blue: Probability $\Pr\{B(N) > R_{\max}\}$ of exceeding LTE data rate capacity. Dashed blue: The same probability for 500m-spaced hexagons in the tiling. Green and red: Probabilities $\Pr\{B(N) > 10R_{\max}\}$ and $\Pr\{B(N) > R_{\max}/10\}$ resp. of exceeding LTE Advanced and 3G data rate capacities resp.

simulation in terms of runtime, we remark that in general simulation allows one to look at the full dynamics of the process while sampling produces only static snapshots. In particular, while for noise- and spectrum-based capacity estimation it may be enough to know only the locations of the drones, for a proper conflict avoidance the velocity vectors of conflicting flights are also of importance. In principle, our LiU model may be extended by adding the direction-offlight distribution at every pixel, so that the resulting joint distribution would then be sampled for more sophisticated CD&R scenarios taking into account velocity differences (cf. Remark (1) after eq. (4)); however, such an extension would probably be too computationally demanding. On the other hand, for UAVs which are able to hover, the relative velocities could be less crucial than for manned aircraft deconfliction: direction-oblivious hover-only CD&R might potentially work for sUAS, albeit at the price of reduced efficiency (we are currently quantifying these and related tradeoffs [42]). Another possible extension is to build the drone map in the case when there are obstacles for the UAV flights (geofences). In the presence of geofences, simulations may take even more time due to the need to compute obstacle-avoiding paths (instead of just taking a straightline segments, as in Cal model), which might make our LiU model and sampling even more attractive. Last but not least, it may be interesting to modify the statistical models to include operational constraints like flow control, airways (or the equivalent for sUAS), "smearing" the noise exposure away from points directly under a heavily trafficked path (similarly to how it is done in ATM in the vicinity of airports), etc.

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⁴Other values are possible: e.g., cells of 120nmi in diameter were used in [18]. Note that [18] also viewed $R_{\text{max}}=1.5$ Gbps as "...a very reasonable volume even for current wireless infrastructure – an amount that could be comfortably supported by single cell tower with equipment that is not particularly new.", while our value for LTE capacity R_{max} is two orders of magnitude smaller.

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