

A Threshold Based Airspace Capacity Estimation Method for UAS Traffic Management

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Abstract—This paper provides a mathematical method for airspace capacity estimation. It is motivated by the need to assess the impact of unmanned aircraft systems on low altitude airspace operations. We define capacity as a minimum of metric-specific phase transition thresholds. The definition is flexible to accommodate a wide variety of metrics defined for the airspace and hence, can be used to compare different unmanned traffic management system approaches. We provide a proof of concept using a metric based on the size of de-confliction problems. The probability of occurrence of large conflicts show phase transition as the traffic density is increased. The traffic density at phase transition i.e. the metric-specific capacity measure, increases with decreasing minimum separation tolerance. Traffic management systems which allow for a higher proximity between aircraft should therefore improve the airspace capacity. Further work must incorporate a wider range of metrics and "sense and avoid" algorithms for a more rigorous validation and application of our airspace capacity estimation method.

Keywords—Capacity, Airspace, unmanned aviation, UTM

I. INTRODUCTION

Airspace is utilized today by far lesser aircraft than it can accommodate. Several 'self-separation' design concepts and decentralized control strategies that transfer some of the separation responsibility to the cockpit have been proposed to increase this aircraft volume[1]–[3]. However, a theoretical approach to airspace capacity for measuring any such increase is not well elaborated. Conventional approaches using air traffic complexity measures such as Monitor Alert Parameter (MAP)[4] or Dynamic Density (DD)[5,6] based on controller workload become less relevant with the advent of automated small-unmanned aircraft systems.

The low altitude uncontrolled airspace primarily used by general aviation is further underutilized owing to the low manned Visual Flight Rules (VFR) air traffic densities. The next phase of unmanned aviation with Beyond Line Of Sight (BLOS) operations is expected to fill that same airspace with traffic, orders of magnitude higher. So, how many aircraft can the airspace accommodate under a given set of technological capabilities, operation requirements, protocols and conditions such as safety, stability and noise levels?

We seek to answer the question with a theoretical definition of airspace capacity that is derived from the metrics defined on the airspace. The metrics account for the variety in the conditions, necessary to be satisfied simultaneously. As these

conditions would change based on the traffic management system used, our definition also allows for comparison of different control strategies. For example, if two decentralized airspace concepts have airspace capacities greater than the expected demand of Unmanned Aircraft System (UAS) traffic, our definition allows to measure which has the higher capacity and hence is preferable.

Existing approaches to measuring airspace capacity are designed to assess manual controller workload. The future air traffic control (ATC) may be a mix of both manual and automated with most of the recent approaches inclined towards the latter. Capacity definition should therefore be less dependent on the nature of ATC although it may help assess the choice of the type of control, e.g. - centralized vs decentralized control. Attempts at such a definition have been made but could be improved further. Section II presents a discussion on the past approaches to airspace capacity and explains the arguments further.

Our definition of airspace capacity and the estimation method are presented in Section III and IV respectively. For a given metric, there is a certain acceptable value up to which the airspace is considered operable. The number of aircraft at which the metric attains that acceptable value is the metric-specific capacity. Given a set of metrics with respective acceptable limits, the minimum of all the metric-specific capacities evaluated is then the airspace capacity. We further refine our generic definition to account for convergence to the acceptable metric values in probability. In other words, the airspace has exceeded its metric-specific capacity if the metric exceeds its acceptable limit with high probability. The metric-specific capacity is the threshold at which this probability phase transition occurs.

As a first application of our definition, we choose the size of the largest de-confliction problem that arises for a given traffic density as our metric. Conflict is simply defined as a loss of minimum separation. The allowable limit of the metric is based on aircraft conflict avoidance literature and air transportation practice. As many as 100,000 unmanned flights a day are expected in a metropolitan region [7]. We therefore simulate this traffic for two separate metropolitan regions, namely the San Francisco Bay Area in the US and Norrköping municipality in Sweden and evaluate the airspace capacity for each region based on the phase transition thresholds. This sample application is described in detail in section V.

Preliminary results are presented in section V-C.

A wider application of our definition would require the use of several metrics and more complex simulations that utilize proposed and practiced conflict resolution algorithms. Section VI concludes this work with a discussion of ideas for the same and proposed extensions of the work.

II. LITERATURE REVIEW

The capacity of an airspace can be fundamentally understood as the maximum number of aircraft that it can safely accommodate. Capacity estimation approaches in literature evaluate this safety from controller and pilot workload under a given set of constraints [8]–[11]. They base this on air traffic complexity measures such as MAP, the maximum number of aircraft an ATC controller can handle at any given time and DD, a weighted sum of factors that affect the air traffic complexity. Furthermore, they are defined based on an assumption of a structured airspace and air traffic management (ATM) that includes monitors, sectors and airways.

However, first of all, manual controller workload becomes less relevant as the industry moves towards automation. Dynamic density definition also varies in terms of factors included and weights assigned to them and therefore does not have a single agreed upon model in literature [12]–[14]. Hence, any capacity estimation based on such measures becomes even less relevant. Second, unmanned aviation needs unmanned traffic management to the extent possible. A proper capacity definition should therefore not be restricted to the type of control, manual or automated. Finally, UAS traffic will inhabit an airspace that may or may not be structured. So, the airspace capacity should also not be restricted to an assumption of structure.

Future UAS operations may be free flight by nature i.e. individual flights could prefer responsibility for determining their own courses independent of a global plan or system. UAS Traffic Management (UTM) should therefore support user preferred flight trajectories to the extent possible. ATM architectures with the same objective for manned free flight were researched by Bilimoria et. al at NASA as part of their Distributed Air/Ground Traffic Management (DAG-TM) concept[2,15]–[17]. DAG-TM is characterized by distributed information sharing, decision-making and/or responsibility among a triad of agents: the Flight Deck (FD), Air Traffic Service Provider (ATSP), and Airline Operational Control (AOC). From a UTM perspective, this is analogous to the on board autopilot (FD), the UTM service provider (ATSP) and the UAS operator/command center (AOC).

The above NASA work presents another approach to airspace capacity that can compare distributed and centralized approaches[16]. It is independent of structure or controller workload. For a given representative area at a particular altitude, the utilized airspace is the percentage of that area covered by aircraft with a defined protected zone around them. It establishes a 90.7% theoretical capacity on airspace utilization, assuming a minimum separation of 5 nautical miles and under the consideration of a unidirectional uniform speed packing of the aircraft (Figure 1). Such an approach is dependent on the reference area selected and unidirectional consideration means not everyone gets from origin to destina-

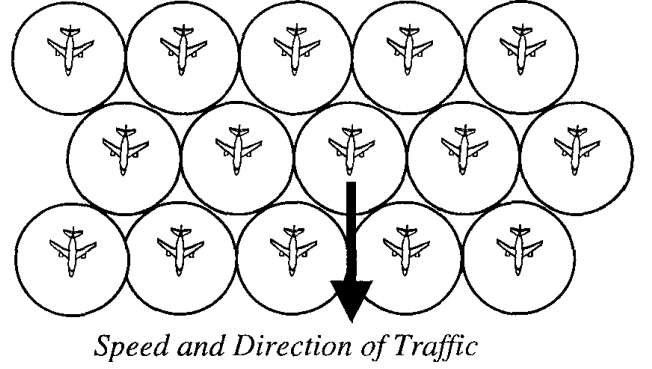


Fig. 1: Optimal packing allowing only one flow direction [16]

tion. Hence, this capacity definition, although quite insightful, is yet restrictive.

Our definition of airspace capacity utilizes all these insights and is based on metrics defined for an airspace. The DAG-TM literature gives some types of metrics that can be used to apply our capacity definition to evaluate any UTM architecture for free flight, namely - Performance, Safety and Stability. We next present our formal definition of airspace capacity. A discussion of how the above metrics can be incorporated in the capacity estimation is covered in our proposed extensions of the current work.

III. CAPACITY DEFINITION

A metric M is a family of random variables parametrized by an integer which in our case represents the expected number of aircraft. The metric evaluated for a specific integer n is denoted by $M(n)$. We require that M is non-decreasing, i.e. for $n' > n$, $P\{M(n') < M(n)\}$ tends to 0 as n increases (i.e. $M(n')$ majorizes $M(n)$).

For an airspace, given a metric M with an acceptable level M' , the metric-specific airspace capacity is the number of aircraft N , such that an additional aircraft makes the probability that the metric exceeds its acceptable level, very high. In other words, the number of aircraft N is the *metric-specific airspace capacity*, if for any small $\epsilon > 0$ and some $\eta \in (0, 0.5)$, $P\{M(\lfloor N - \epsilon \rfloor) > M'\} < \eta$ and $P\{M(\lceil N + \epsilon \rceil) > M'\} > 1 - \eta$.

Typically, more than one metric must be simultaneously considered when evaluating the capacity of an airspace. Hence, we expand our definition over the set of metrics under consideration.

Let $M_S = \{M_1, M_2, \dots, M_k\}$ be the set of metrics defined on a set of aircraft A for a given airspace. Each M_i , $i \in [1, k]$, is evaluated for a given number of aircraft n and must be a non-decreasing function as per our convention. Let M'_i , $i \in [1, k]$, be the acceptable levels of the corresponding metrics. Then we can define the number of aircraft N_i as the *metric-specific airspace capacity*, if for any small $\epsilon > 0$ and some $\eta \in (0, 0.5)$, $P\{M_i(\lfloor N_i - \epsilon \rfloor) > M'_i\} < \eta$ and $P\{M_i(\lceil N_i + \epsilon \rceil) > M'_i\} > 1 - \eta$. The overall *capacity of the airspace* is then $N = \min(N_i)$.

This airspace capacity definition includes deterministic cases. For example, let us evaluate the capacity N of a holding airspace around an airport. Let the metric M be the negative of average miles in trail separation between the aircraft in the

holding pattern. For simplicity, we will assume the holding airspace is a circle and two aircraft can only be in sequence and not next to each other. For a given number of aircraft n , $M = -C/n$, where C is the circumference of the circle. Let the acceptable average separation be M' . Then the capacity of the holding airspace is $N = \lfloor -C/M' \rfloor$. If the number of aircraft in the holding pattern is even one less than N , $P\{M > M'\} = 0$. If the number of aircraft in the holding pattern is even one more than N , $P\{M > M'\} = 1$.

The above example shows a sharp transition in probability. N is therefore defined as a sharp phase transition threshold. The above formulation is therefore a strong form definition of airspace capacity. It forces N to be a single number. For more complex metrics, this N needs to be evaluated based on air traffic simulation that computes the metrics by varying the traffic densities. As the number of aircraft n become large, the airspace capacity may not necessarily be a single number but a transition range instead. A weak form of the definition would allow N to be a phase transition range.

Suppose, each N_i is a range $[N_{i,l}, N_{i,r}]$. Then N_i is the *metric-specific airspace capacity range*, if for any small $\epsilon > 0$ and some $\eta \in (0, 0.5)$, $P\{M_i(\lfloor N_{i,l} - \epsilon \rfloor) > M'_i\} < \eta$ and $P\{M_i(\lceil N_{i,r} + \epsilon \rceil) > M'_i\} > 1 - \eta$. The overall *capacity range of the airspace* is then $N = [\min(N_{i,l}), \max(N_{i,r})]$.

IV. CAPACITY ESTIMATION METHOD

In the next section, we describe a sample application of our capacity definition by abstracting the airspace as a graph. It is noteworthy that since our definition presents capacity as a phase transition threshold, phase transition results from the theory of random geometric graphs apply directly, if the graph nodes are chosen uniformly at random and the metric does not decrease when edges are added to the graph.

In detail, let $n \in \mathbb{N}$, $r \in \mathbb{R}^+$ be an integer and a non-negative number respectively. A *random geometric graph (RGG)* is a graph whose nodes are n randomly placed points in a given region, and whose edges connect two vertices u and v if and only if the distance between them is less than r : $|uv| < r$.

A *property* Π (of a graph) is formally defined as a subset of all graphs on n vertices. For instance, *connectedness* is a property that contains all connected graphs, *tree* is a property that contains all trees, *having a connected component of size at least 17* is a property that contains all graphs that have a connected component of size at least 17, etc. (This formalism is a bit heavy: it is much more intuitive and common to speak about “a connected graph” instead of “a graph belonging to the *connectedness property*”, about “a tree” instead of “a graph belonging to the *tree property*” and so on).

A property is called *monotone* if adding edges to the graph does not break the property, or formally, if the graph with added edges still belongs to the property. Thus, connectedness and having a connected component of size at least 17 are monotone, but tree is not.

Now, consider RGGs for different values of r . As r increases, the graph gets “more and more edges”: if $|uv| < r$, then also $|uv| < R$ for any $R > r$. A fundamental result in the RGG

theory [18] says that any monotone property Π of RGG has a “sharp threshold” r_Π : for r that are smaller than the threshold, the probability of observing a RGG with the property Π is small; on the contrary, for r above the threshold, the RGG will have the property with probability close to 1.

Our definition of airspace capacity is inspired by this result. The snapshot of an airspace at a given time can be abstracted as a graph. The aircraft are the vertices of the graph (Figure 3). Aircraft in conflict are connected by an edge. So, the first capacity estimation method is to use threshold results on such a graph straight away to estimate airspace capacity provided the associated assumptions are satisfied.

However, as is the case with the sample application discussed next, the associated assumptions may not always be satisfied. For example, threshold existence for monotone properties is only proven under the assumption of a uniform distribution over the randomly located vertices. Unmanned traffic may not necessarily be uniformly distributed. In such a scenario, the second method of capacity estimation is to simulate the traffic and the range where the probability phase transitions for the chosen metrics occur. Our sample application shows an example of this second estimation method.

V. SAMPLE APPLICATION

For an application of our generic capacity definition to low altitude airspace with unmanned air traffic, we use the following metric: the size of the largest de-confliction problem observed for a given traffic density. We assume a minimum allowable separation between two UASs. The measure of the airspace capacity is therefore the range, where the probability that the computed metric exceeds the acceptable size of the largest de-confliction problem, shows a phase transition. We investigate how the metric behaves w.r.t. the traffic intensity and the conflict radius (the minimum allowable separation). The formal definitions follow.

A. Metric Definition

We use the model and approach from [7] and consider UAS with strictly vertical takeoff and landing, flying on a fixed flight level as shown in Figure 2. All aircraft are at the same level because with an under 400ft restriction on commercial UAS operations[19], there is very little room for multiple levels. Thus, our setup is two-dimensional – any conflict between the drones may happen only due to the loss of minimum lateral separation.

To determine the size of the de-confliction problem, we follow a cluster based analysis as defined and discussed by Durand[20] and Bilimoria[21]. A snapshot of the airspace is abstracted as a graph (Figure 3). The vertices of the graph are the aircraft, and two vertices are connected when the aircraft are in conflict, i.e. when the distance between them is smaller than an allowable minimum separation r . Each connected component of the graph thus represents a set of aircraft to be jointly de-conflicted (called a cluster). The cardinality of this set is the number of vertices in the component, or the size of the de-confliction problem (cluster size). Our metric is the size of the largest de-confliction problem observed in the snapshot i.e. the size of the largest component.

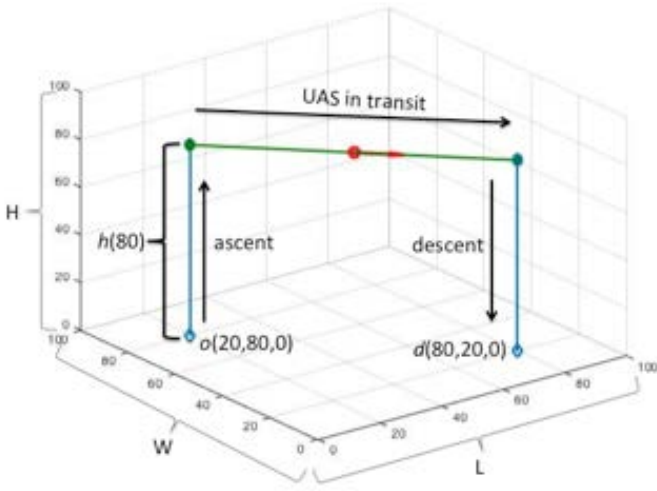


Fig. 2: A typical UAS flight path

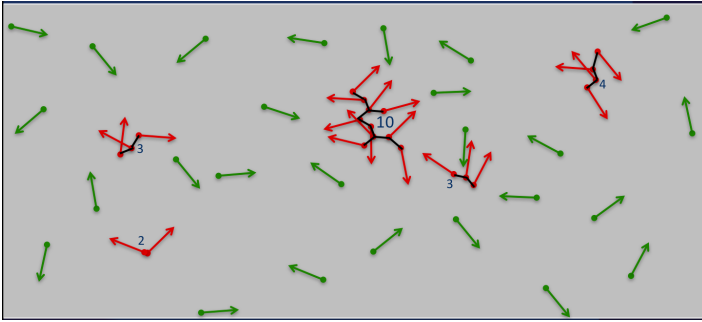


Fig. 3: Abstraction of an airspace snapshot as a graph with cluster sizes indicated. (Aircraft in conflict are shown by red dots. The conflict pairs are connected by a black line. The arrows indicate the heading of the aircraft.)

Since transportation practice tends to eliminate all free de-confliction problems with more than three vehicles, we choose the acceptable size of the largest de-confliction problem as 3 and use de-confliction problem size greater than 3 as the definition of large. De-confliction literature as elaborated in [7] shows that this number might improve in future for automated real time resolutions.

B. Simulation

We use the simulation setup from [7]. The airspace is modeled as a cuboidal volume LWH defined by a rectangular area extruded to a given height H . Each UAS is a quadruple (o, d, h, t) i.e. it has an origin, destination, height and start time. A typical flight was shown in figure 2. The flights' origins and destinations were generated randomly based on the population density over the rectangular area. This preserves the actual shape of the geographical area and the volume of airspace used.

The total number n of flights expected during the day was given, and the intensity of the traffic starting or ending at a point p of the domain was proportional to the population density at p (that is, the starting times of the flights from p form a Poisson process with the rate proportional to the density). The simulations were run in Matlab for two regions:

Bay Area in the US (Figure 4) and Norrköping municipality in Sweden (population density data courtesy of [22]). In each of the regions, we simulated 6 days of traffic, varying n from 10 to 200,000 flights a day and r from 5m to 300m. We used the size of the largest component in the conflict graph as the measure M , and set $M' = 3$.

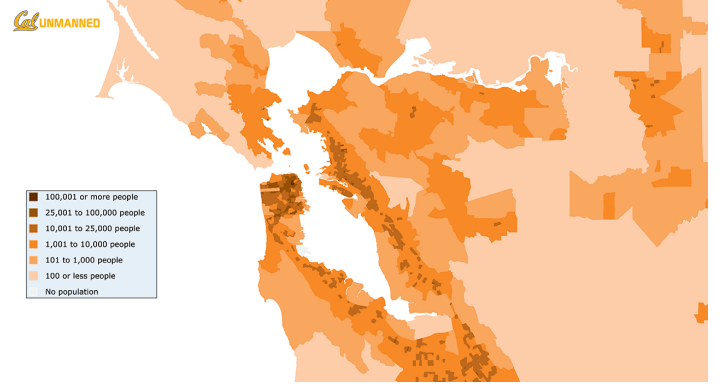


Fig. 4: Population Density Map of the Bay Area [23]

C. Results

Figure 5 shows the probability $P\{M > M'\}$ of observing large de-confliction problems (clusters of size greater than 3) when the conflict distance is $r = 300m$. It can be seen that the probability goes from 0 to 1 around $n \sim 30000$ for Bay Area (left) and around $n \sim 15000$ for Norrköping (right). These numbers define the capacities of the airspaces over the regions (under flight level assumption as per V-A).

Naturally, it is possible that with the advance of the conflict detection and resolution techniques and hardware, the conflict threshold r will decrease from 300m (which was used in our simulations above and also in [7]). We therefore experimented also with the whole range of values for $r < 300m$. Further figures show the probability distributions $P\{M > M'\}$ for the different cases considered: Figures 6, 7 show the results for Bay Area, and Figures 8, 9 – for Norrköping.

It is noteworthy that Figures 7, 9 actually show the entire band of capacity range in light blue and green over all the combinations of the minimum separation and traffic densities chosen. The above mentioned single capacity number are the centre values of the respective capacity ranges. The airspace capacity in general increases with decreasing minimum separation tolerance.

VI. CONCLUSION AND FUTURE WORK

Our sample application results show that the probability of observing large de-confliction problems exhibit quite sharp thresholds at the airspace capacity. Modes of operations with (n, r) in the blue areas on Figure 7 (right) and Figure 9 (right) are very unlikely to exceed the capacity, while operating in the yellow areas will almost surely exceed the capacity leading to large de-confliction problems. The (thick) regions that separate blue from yellow show the relations between the "critical" traffic intensity and conflict radius. We believe that the graphs like these will help the authorities in quantifying the tradeoffs between the allowable density of the UAV traffic (n) and the conflict detection and resolution capabilities (r).

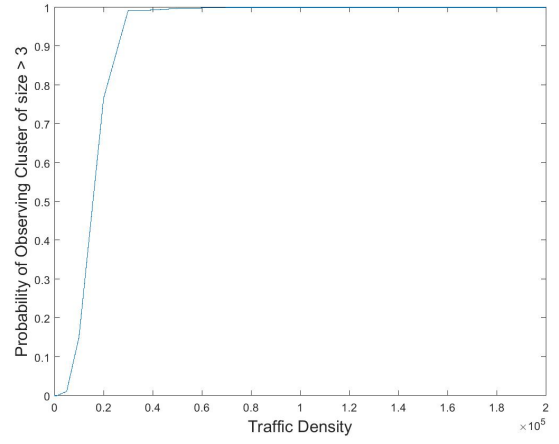
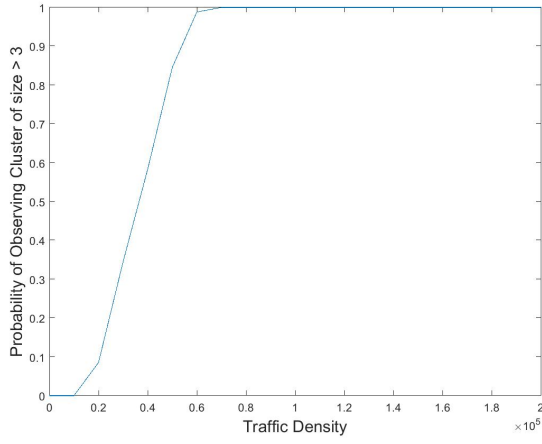


Fig. 5: $P\{M > M'\}$ as function of n (for $r = 300m$). Left: Bay Area. Right: Norrköping.

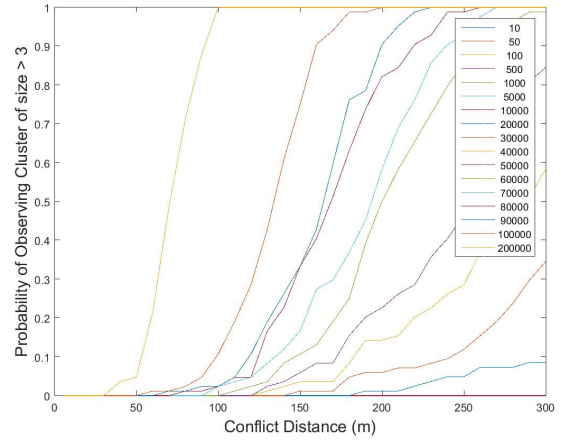
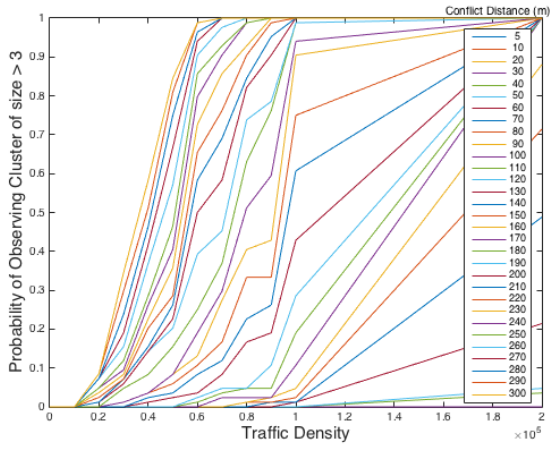


Fig. 6: Bay Area. Left: $P\{M > M'\}$ as function of n for various r . Right: $P\{M > M'\}$ as function of r for various n .

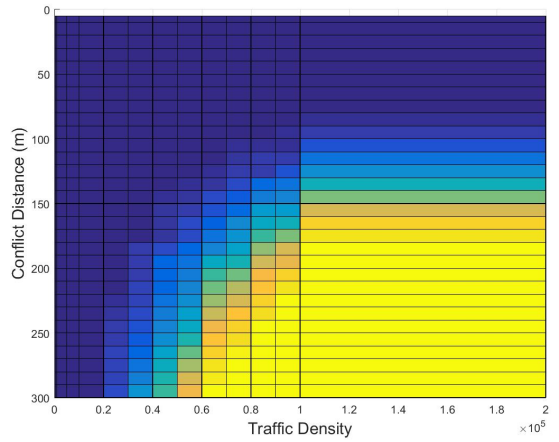
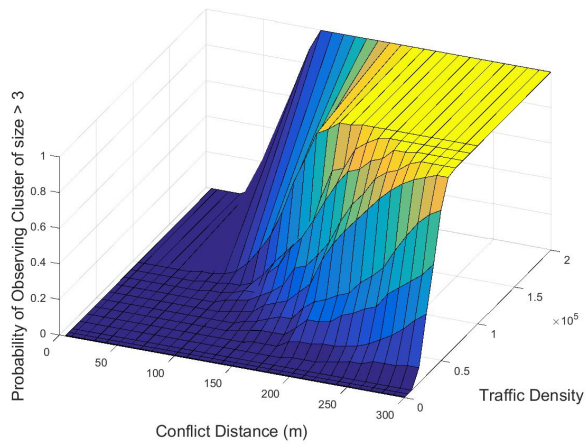


Fig. 7: Left: $P\{M > M'\}$ as function of both n and r , right); Bay Area. Right: The top view of the graph.

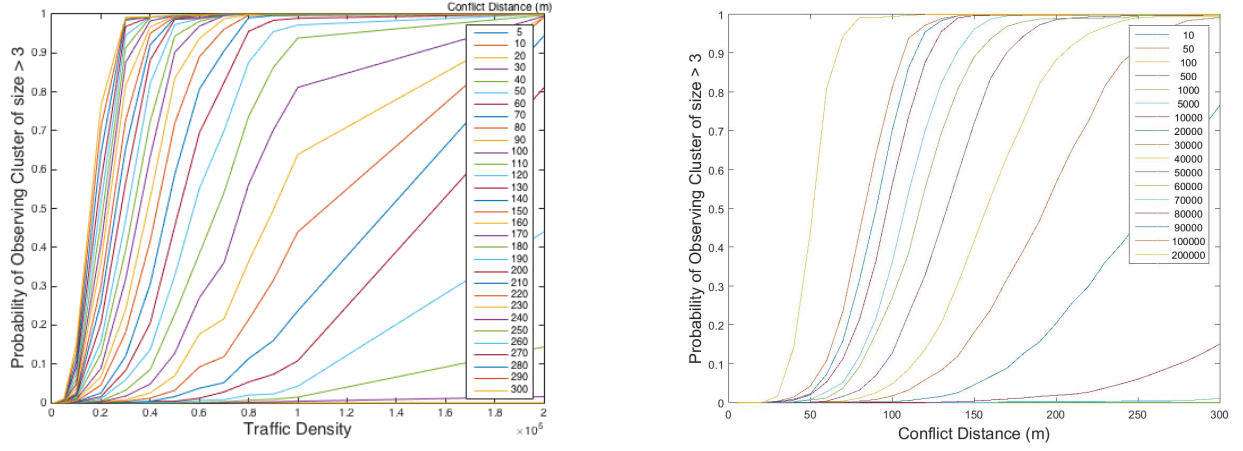


Fig. 8: Norrköping. Left: $P\{M > M'\}$ as function of n for various r . Right: $P\{M > M'\}$ as function of r for various n .

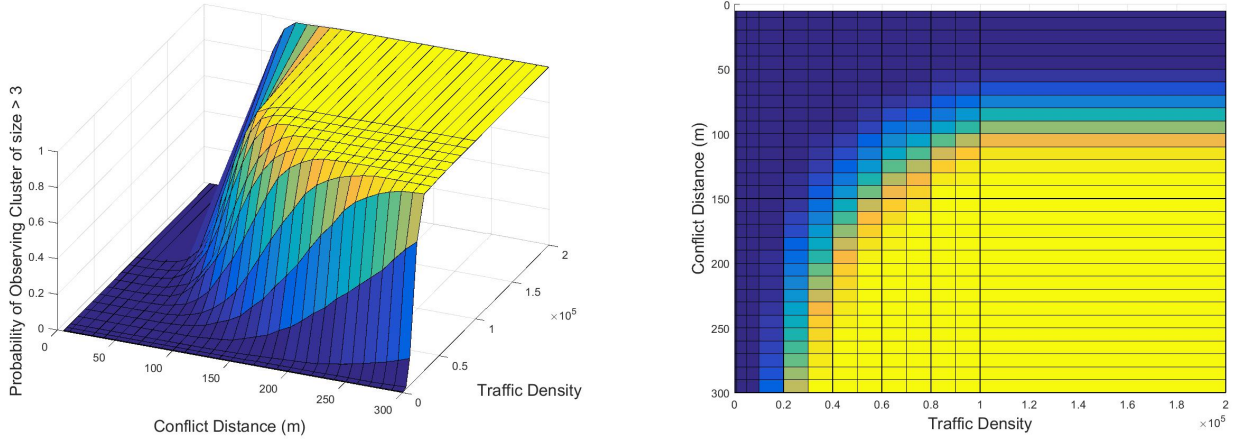


Fig. 9: Left: $P\{M > M'\}$ as function of both n and r , right); Norrköping. Right: The top view of the graph.

It is of interest to estimate the airspace capacity for other values of allowable limits M' , and more generally – under a wider range of metrics M . The current sample application is based on a simple assumption of conflict as loss of minimum separation and doesn't include any conflict avoidance algorithm as part of the simulation. However, taking heading into consideration and using a conflict avoidance technique widely accepted by the aviation community would lead to better bounds on the airspace capacity.

One of the most important concerns that the UTM community is currently facing is to measure the volume of unmanned aircraft that can be accommodated in the existing airspace based on considerations of system safety, system performance, spectrum required for communication and noise levels. Our definition enables us to respond to that concern. Future extensions of this work will include stricter results on aircraft capacity based on system safety considerations with aircraft heading and conflict resolution accounted for. Airspace capacity estimates based on the other aforementioned considerations will also be explored in future work on extended applications of our airspace capacity definition.

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