Capacity Estimation for Low Altitude Airspace

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This paper uses a threshold based mathematical definition to estimate capacity for future sUAS traffic in low altitude uncontrolled airspace based on safety and performance considerations. It is motivated by the need to assess the impact of large-scale close proximity unmanned aircraft operations on communities and existing manned airspace. We simulate unmanned traffic over urban areas and estimate metrics focused on safety and performance efficiency. The effect of increasing traffic density on the metrics shows that safety is potentially the most critical capacity determining factor of the two.

I. Introduction

The airspace today is used by far lesser aircraft than it can accommodate. The next phase of unmanned aviation with Beyond Visual Line Of Sight (BVLOS) operations is expected to fill that same airspace with traffic, orders of magnitude higher. So, how many small unmanned aircraft can the existing low-altitude airspace accommodate under a given set of technological capabilities, operational requirements, protocols and conditions such as safety, stability, performance efficiency and noise levels?

In this paper, we address that by estimating capacity based on two types of metrics – safety and performance efficiency, necessary to be satisfied simultaneously. We extend our threshold based definition[1] of airspace capacity (Section IV) and use a simulation paradigm to establish the results (Section VI). An application of our approach to establish noise capacity appears in.[2] For a given metric, there is a certain acceptable value up to which the airspace is considered operable. The number of aircraft at which the metric exceeds that acceptable value with high probability is the metric-specific capacity, the threshold at which the probability phase transition occurs. Given a set of metrics with respective acceptable limits, the minimum of all the metric-specific capacities is then the airspace capacity.

There is no representative small Unmanned Aircraft System (sUAS) traffic in the airspace today to base our capacity numbers on. Hence, we start by considering sUAS flights per day in a metropolitan region as our measure of capacity following prior analysis in[3] that suggests 100,000 flights per day based on package delivery. We present the sample application of our capacity definition for the San Francisco Bay Area in the United States and the Norrköping municipality in Sweden, considering upto 200,000 flights per day. Finally, we establish results for Safety and Performance Efficiency only for the Bay Area.

The airspace capacity for unmanned traffic will be a function of the technology applied to it. This technology will have a collision detection, resolution, and avoidance (CDRA)[4] component on the network edge, i.e., on-board each UAS, and Unmanned Aircraft System (UAS) Traffic Management (UTM)[5] component in the network core (the cloud). To establish the feasibility of a volume of unmanned traffic, we need to pick the metrics, use parameters that model the technology, and develop a computational process that puts numbers to the metrics, as a function of the technology parameters.

In Section III we discuss the metrics considered for sUAS traffic and required to be satisfied simultaneously. To model technology, we make the following assumptions. We consider sUAS with strictly Vertical Take Off and Landing (VTOL) capability. They take off, fly directly from origin to destination and land. This models the evolution of the system if everyone was allowed to fly their most preferred route. All sUAS fly at a uniform speed at the same level because with an under 400ft restriction on commercial sUAS operations[6] in

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urban areas, there is very little room for multiple levels. Thus our setup is two dimensional and any conflicts and collisions may happen only due to loss of minimum separation.

However, the resolution can use the third dimension. Compared to regular aircraft, sUAS are very small. We model this by allowing very close collision proximity (from 2.5m (best) to 20m (worst)). They are also highly maneuverable. Hence, for conflict resolution, we use vertical avoidance by modelling altitude control within a range such that the horizontal velocity is maintained. We aim at establishing capacity for the most basic system with minimal feasible assumptions. Hence, we ignore sensor and navigational uncertainties (such as deviations from trajectory, delays in aircraft detection, etc), static and dynamic obstacles (e.g., buildings and wind) and air worthiness violations (aircraft failures).

Finally, our computational process is a simulator described under V-B. The structure of the paper is as follows. We first present a review of related work that motivates this paper under section II. Our metrics and Capacity definition with a sample application are presented under sections III and IV respectively. The conflict detection and resolution algorithm used, the simulator requirements and details of the sample simulator used are discussed in V. In VI we discuss our capacity estimates based on Safety and Performance efficiency for Bay Area. The results also focus on further insights from the distributions of the efficiency numbers to emphasize the effect of lack of management on fairness in the system. Safety turns out to be the bottleneck with current requirements on the system. Further research directions to increase capacity or establish tighter bounds by modeling more airborne technologies form the subject of section VII which also summarizes the findings of the paper.

II. Literature Review

The simplest notion of airspace capacity is the maximum number of aircraft that can traverse an airspace in a given time under a set of requirements. Capacity estimation approaches in literature evaluate this from controller and pilot workload.[7–10] Capacity is derived from air traffic complexity measures such as Monitor Alert Parameter (MAP),[11] the maximum number of aircraft an Air Traffic Control (ATC) controller can handle at any given time and Dynamic Density (DD),[12,13] a weighted summation of factors that affect the air traffic complexity. These are defined based on an assumption of a structured airspace and Air Traffic Management (ATM) that includes monitors, sectors and airways.[14–16] Capacity is then estimated using fast time and real time simulation methods.[17] These estimates are highly subjective owing to the dependency on manual controller judgment during the experiments, who are also assumed to be the bottleneck of the system. Further, even if confined to a physical space, a good capacity estimation approach should account for structure instead of being restricted by it.

The Eurocontrol Care-Integra is another novel approach that addresses this by modeling the ATM system as a combination of several information processing agents, each with an associated information processing load (IPL).[18] The system reaches capacity when one of the agents overloads. This overload threshold is easy to determine for machine agents but needs subjective judgment for human agents. However, this approach determines the bottleneck in the system instead of assuming it. Our capacity estimation approach is therefore inspired by this. We base our capacity on the metrics themselves that need to be satisfied. The metrics and their acceptable limits account for the type of controller and hence the approach can handle a hybrid management – human, automated or both.

Future sUAS operations may be free flight by nature i.e. individual flights could prefer responsibility for determining their own courses independent of a global plan or system. UTM should therefore support user preferred flight trajectories to the extent possible. Any chosen metrics should account for this. ‘Self-separation’ design concepts and decentralized control strategies that transfer some of the separation responsibility to the cockpit have been proposed for manned aviation.[4,19,20] ATM architectures with the same objective for manned free flight were also researched by Bilimoria et al. at NASA as part of their Distributed Air/Ground Traffic Management (DAG-TM) concept.[19,21–23] DAG-TM is characterized by distributed information sharing, decision-making and/or responsibility among a triad of agents: the Flight Deck (FD), Air Traffic Service Provider (ATSP), and Airline Operational Control (AOC). From a UTM perspective, this is analogous to the on board autopilot (FD), the UTM service provider (ATSP) and the UAS operator/command center (AOC).

Krozel et al.[22] show that following types of metrics can be potentially used to evaluate any UTM architecture for free flight (the sample measures used[22] for manned ATM are listed in parenthesis): Safety (number of actual conflicts and conflict alerts), Performance (Change in direct operating cost), Stability (number of forced conflicts (domino effect)) and DD (aircraft density, average proximity and average point of closest approach). Of these, we focus on the first two for sUAS traffic in this paper. Further a recent MITRE report[24] proposes a maximum loss of 1 sUAS flight per 1000 flight hours over urban areas. Hence, this forms the basis of our chosen metrics described in section III.

Next comes the choice of a CD&R algorithm. CD&R methods[4] in aviation literature have been primarily developed for large aircraft flying at higher altitudes and lower densities than the expected future sUAS traffic. An example of a simple rule as used by Bilimoria et al.[22] is shown in Figure 1. However, owing to their size and maneuverability, sUAS provide a unique opportunity for simpler conflict resolution algorithms. Proposed future sUAS operations[25] might be done primarily by aircraft that have VTOL capability and high vertical acceleration rates. Hence, a simple velocity or altitude control is worth studying for safety of sUAS traffic. In section V, we present a CD&R algorithm based on altitude control.

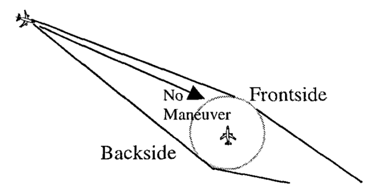

Figure 1. CD&R geometry based on choosing the lower cost choice between the frontside and backside maneuver[22]

Given the metrics and a CD&R algorithm, we next need to measure the low altitude airspace capacity. We have proposed a capacity estimation approach for sUAS traffic based on phase transition thresholds of chosen metrics beyond their allowable limits.[1] In this paper, we extend that work by producing results for safety and performance metrics. The capacity definition is reproduced in section IV.

Finally, we need a simulator that can simulate sUAS traffic densities so that the allowable limits of chosen metrics are reached with reasonable confidence. Many advanced simulation and evaluation tools developed for ATM (like BlueSky, TMX, ACES, AEDT, FACET) are overly complicated for our purposes – they take into account interaction with a variety of actors (air traffic controllers, the military, etc.) that are an overkill for the study of fundamental low-altitude UTM questions of traffic behavior and capacity.

Depending on the development of UTM, some of the factors present in ATM simulators (e.g., hazardous weather, community effect via noise and pollution, existence of no-fly zones) should be simulated/evaluated also in UTM, as the research progresses. Naturally, there are also features that are not relevant for ATM (and therefore are rightly absent from the ATM simulators) while being of high importance to UTM. One example of such a factor is geo-data on near-ground obstacles, public and private land ownership and rights of way. Unlike conventional aircraft which stay primarily above 1000 feet in urban areas, except near airports, future sUAS operations are expected to occur below 500 feet. Simulation of safety from near ground static and dynamic obstacles and avoidance of exclusion/non-permitted flying zones therefore become necessary components of a sUAS traffic capacity generating simulator.

Lastly, ATM and its simulators are focused mostly on scheduled and deterministic traffic. Aircraft take off and land in distinct and well defined areas. For UTM, this will change in future when, in the words of Dr. Kopardekar,[26]"every home will have a drone and every home will serve as an aerodrome". Given the diversity and unpredictability (temporal and spatial) of its operations, randomness is a much bigger player for future UAS traffic. To include this stochastic component, we look at some past approaches. A probabilistic setup which can be called *Dutch model* was used in PhD thesis of Hoekstra,[27] developed by Jardin,[28] and more recently explored within the Metropolis project by TU Delft.[29] In this model the aircraft are distributed *uniformly* in the given airspace. The direction of flight is also uniformly distributed in 0...360°; in[29] the different direction cones are separated by altitude. This uniform spatial distribution may not necessarily translate to the UAS traffic.

We instead use a population density model in this work. Flights' endpoints are sampled from the population density (neither the vehicles locations nor their headings are distributed uniformly) in line with the 'every home aerodrome' vision.[26] As in the basic version of the Dutch model, the flights occupy a single level, so the setup is essentially two-dimensional while the resolution uses the third dimension (see current restrictions on operations under 400ft;[6] see also Tompa et al.[30] for the "horizontal-maneuvers" TCAS work).

III. Metrics

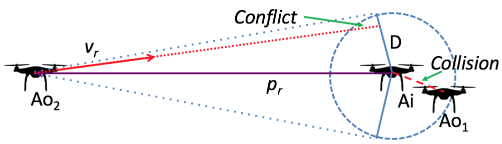

Figure 2. Conflict and Collision. Ao **- Own sUAS,** Ai_1 **&** Ai_2 **- Intruder sUAS. The aircraft are shown in relative frame of reference**

We first define the notion of a collision and conflict. Any sUAS should stay out of a minimum separation exclusion zone (a cylinder with radius D and height D) around another sUAS (Figure 2 shows the top view. The vertical cylinder is shown in figure 8). A *collision* is the loss of this minimum separation between any two sUAS. Given their projected paths in the horizontal plane, if an sUAS will eventually enter within the minimum separation of another sUAS, the two aircraft are in *conflict*. We describe this mathematically in section V.

Based on the above definitions, we use two metrics for estimating the low-altitude airspace capacity for sUAS traffic in this work:

A. Safety

Safe operation of the airspace is of utmost importance. Following the proposed requirements by MITRE, we propose the necessary safety metric as the *Total Loss of Flight per Flight Hour* with an acceptable limit of 0.001 as suggested in.[24]

B. Performance

Safe operation might often result in loss in efficiency due to longer travel distances and times which then translate into higher operating costs (fuel, wear, etc.) and hence lower performance. We therefore use the *Change in Direct Operating Cost* as proposed by Krozel et al.[22] as our performance metric. However, this metric considers the added effect of Cost for Extension of Travel Distance and Travel Time. Since there is no extension of travel time as sUAS do not change their horizontal speed in the simulation, we modify it in the present work just to *Percentage Extension of Travel Distance* with an acceptable limit of 10%.

We use the above metrics to determine capacity. These can be further extended as research in the field progresses. One example of an additional metric is noise level which has been studied in.[2,31]

IV. Capacity Definition

A metric M is a family of random variables parametrized by an integer which in our case represents the expected number of aircraft. The metric evaluated for a specific integer n is denoted by $M(n)$. We require that M is non-decreasing, i.e. for $n' > n$, $\mathsf{P}\{M(n') < M(n)\}$ tends to 0 as n increases (i.e. $M(n')$ majorizes $M(n)$).

We next state the weak formulation of the capacity definition from.[1] Let $M_S = \{M_1, M_2, \ldots, M_k\}$ be the set of metrics defined on a set of aircraft A for a given airspace. Each M_i, $i \in [1, k]$, is evaluated for a given number of aircraft n and must be a non-decreasing function as per our convention. Let M_i', $i \in [1, k]$, be the acceptable levels of the corresponding metrics.

Let us define the number of aircraft N_i as a range $[N_{i,l},\ N_{i,r}]$. Then each N_i is the *metric-specific airspace capacity range*, if for some $\eta \in (0, 0.5)$, $\mathsf{P}\{M_i(N_{i,l} - 1) > M_i'\} < \eta$ and $\mathsf{P}\{M_i(N_{i,r} + 1) > M_i'\} > 1 - \eta$. The overall *capacity range of the airspace* is then $N = [min(N_{i,l}),\ max(N_{i,r})]$.

This airspace capacity definition includes deterministic cases. A good example is the capacity N of a holding airspace around an airport. Let the metric M be the negative of average miles in trail separation between the aircraft in the holding pattern. For simplicity, assume the holding airspace is a circle and two aircraft can only be in sequence and not next to each other. For a given number of aircraft n, $M = -C/n$, where C is the circumference of the circle. Let the acceptable average separation be M'. Then the capacity of the holding airspace is $N = \lfloor -C/M' \rfloor$. If the number of aircraft in the holding pattern is even one less than N, $\mathsf{P}\{M > M'\} = 0$. If the number of aircraft in the holding pattern is even one more than N, $\mathsf{P}\{M > M'\} = 1$.

The above example shows a sharp transition in probability. N is therefore a sharp phase transition threshold. For more complex metrics, this N needs to be evaluated based on air traffic simulation that computes the metrics by varying the traffic densities. As the number of aircraft n becomes large, the airspace capacity may not necessarily be a single number and hence the *capacity range* definition applies.

A sample application of this capacity definition was studied in[1] with the setup in.[3] The chosen metric (in the absence of any CD&R algorithm) was the size of the largest aircraft cluster (group of aircraft in conflict simultaneously) observed at any given time with an acceptable limit of size 3. The simulation were run by producing traffic in two metropolitan regions – San Francisco Bay Area in US and Norrköping in Sweden, based on their respective population densities (Fig. 3). The traffic densities (n) and the loss of separation distances (r) were varied. We observed that the transfer from conflict-free to unsafe regime indeed exhibits *threshold* properties akin to phase transition (fig. 4): small changes in the input parameters lead to drastic changes in the output. Furthermore, figures 5 and 6 show the entire band of capacity range in light blue and green over all the combinations of the minimum separation and traffic densities chosen.

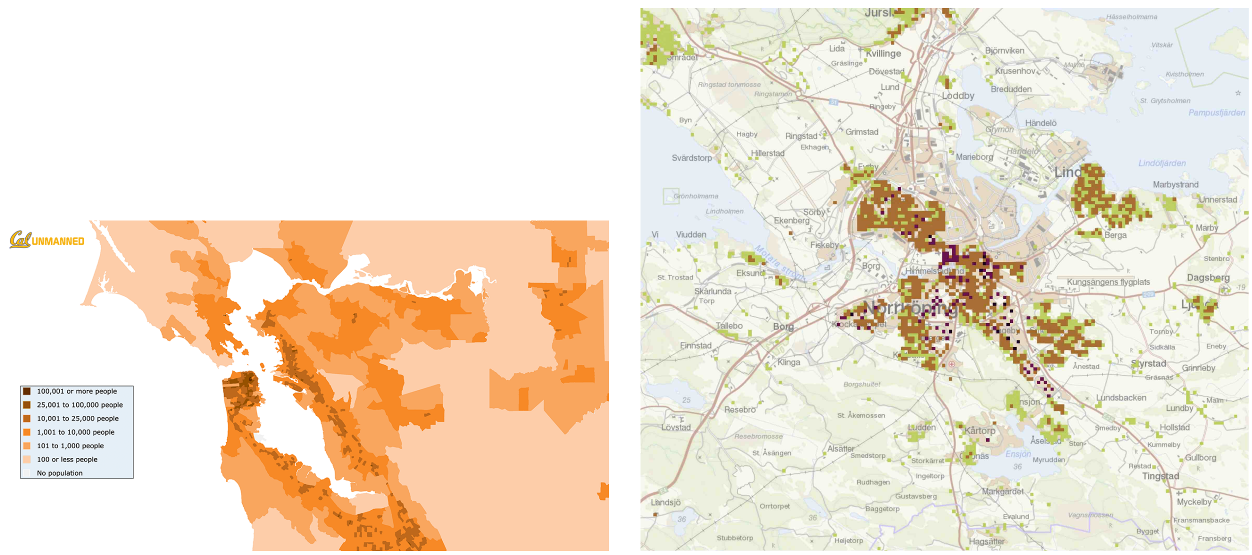

Figure 3. Population Density Map. Left: Bay Area.[32] Right: Norrköping.[33]

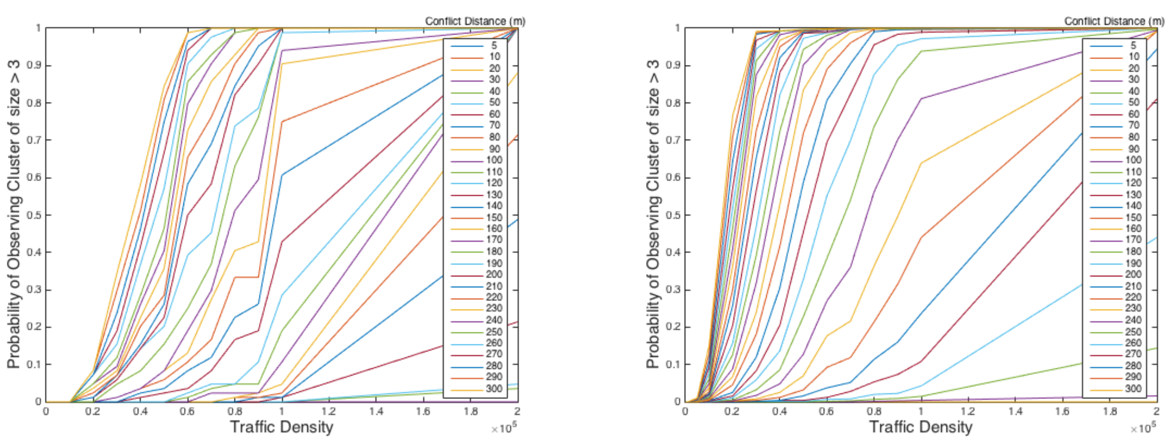

Figure 4. P$\{M > M'\}$ as function of n for various r. Left: Bay Area. Right: Norrköping.

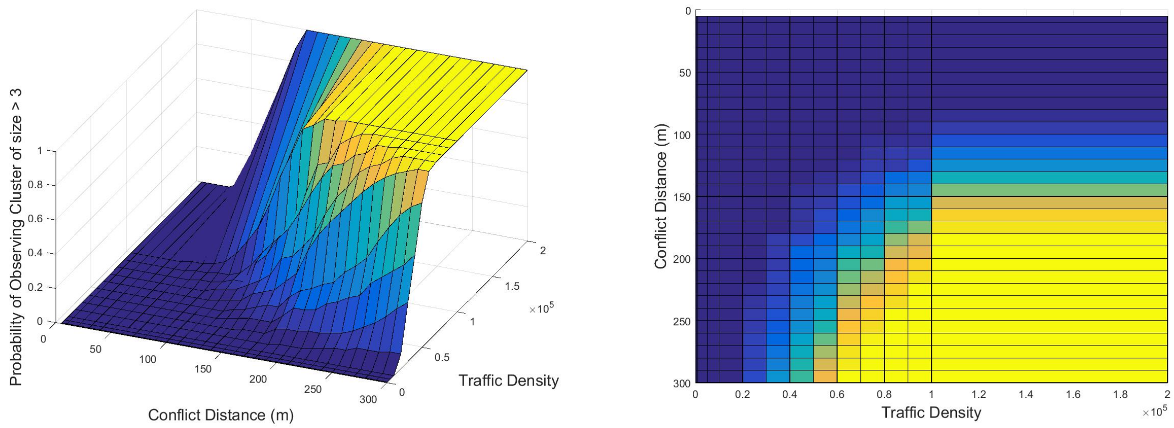

Figure 5. Bay Area. Left: $\mathsf{P}\{M > M'\}$ as function of both n and r. Right: The top view of the graph.

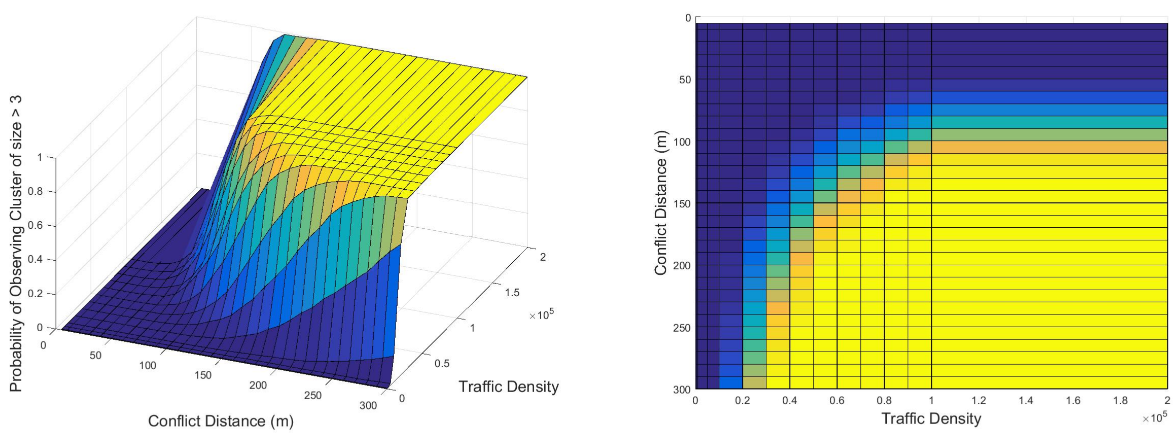

Figure 6. Norrkoöping. Left: $\mathsf{P}\{M > M'\}$ as function of both n and r. Right: The top view of the graph.

V. Capacity Estimation

Now we proceed to expand on the estimation method we use to establish results for Safety and Performance. We first present our CD&R algorithm for sUAS followed by a discussion of the Simulation requirements for sUAS traffic and the description of the simulator used in the current work.

A. Conflict Detection and Resolution

We consider two sUAS A_o (the own sUAS) and A_i (the intruder sUAS), with variables p_o, v_o, u_o and p_i, v_i, u_i respectively having their usual meaning (position, velocity and acceleration control). The kinematic motion of the aircraft is given by

$$\frac{dp_o}{dt} = v_o, \frac{dv_o}{dt} = u_o \tag{1}$$

$$\frac{dp_i}{dt} = v_i, \frac{dv_i}{dt} = u_i \tag{2}$$

$p(t; t_0)$ denotes the solution of the ODE over the time interval $[t_0, t]$ with initial conditions $p(t_0), v(t_0)$. The relative variables with respect to the intruder aircraft are given by

$$p_r = p_o - p_i,\ \dot{p_r} = \dot{p_o} - \dot{p_i} = v_o - v_i = v_r \tag{3}$$

$$v_r = v_o - v_i,\ \dot{v_r} = \dot{v_o} - \dot{v_i} = u_o - u_i = u_r \tag{4}$$

The equations of motion therefore reduce to

$$\frac{dp_r}{dt} = v_r, \frac{dv_r}{dt} = u_r \tag{5}$$

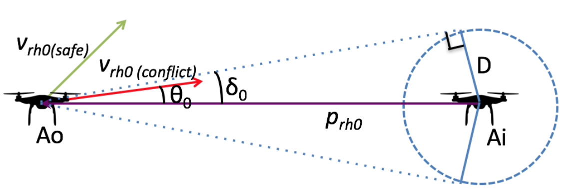

Figure 7. Conflict Detection (Top View). A_o - Own sUAS, A_i - Intruder sUAS. The aircraft are shown in relative frame of reference

We additionally use the following subscripts: h – in horizontal plane and v – vertical direction. Hence, $p_r \in \mathbb{R}^3$ can be written as $p_r = [p_{rh}\ p_{rv}]$. Now, for any interval $[0, \tau]$, we define, $d_{closest,h}(\tau) = min \|p_r(t)\|,\ t \in [0, \tau]$. We also define the cylinder C_i around sUAS A_i as $p \in \mathbb{R}^3$ such that $\|p_{rh}\| \le D$ & $\|p_{rv}\| \le D$. $D \in \mathbb{R}$ is the minimum allowable separation from an sUAS in the horizontal plane and vertical direction.

A_o and A_i are in *collision* at time t if $p_o(t) \in C_i$. A_o and A_i are in *conflict* at time t_0 iff $\exists\ \tau \ge t_0$ such that $d_{closest,h}(\tau) \le D$ & $p_r(\tau) \in C_i$. Then, we call the time of closest horizontal approach – $t_{closest,h}$.

Define,

$$\alpha = Cos^{-1}\frac{\langle p_{rh}, v_{rh}\rangle}{\|p_{rh}\|\|v_{rh}\|} \tag{6}$$

$$\delta = Cos^{-1}\sqrt{1 - \frac{D^2}{\|p_{rh}\|^2}} \tag{7}$$

$$\theta = \pi - \alpha \tag{8}$$

1. *Conflict Detection Condition*

When, $u_{rh}(\tau; 0) = 0$ & $p_{rv}(\tau; 0) = 0$, then for any $t \in [0, \tau]$,

$$\|p_{rh}(t)\|^2 = \|p_{rh}(0)\|^2 + \|v_{rh}(0)\|^2 t^2 + 2\langle p_{rh}(0), v_{rh}(0)\rangle t \tag{9}$$

Further we will drop the (t) and use 0 in subscript to denote value at time $t = 0$ for brevity.

For $d_{closest,h}$, $\frac{d\|p_{rh}\|^2}{dt} = 0$, which gives

$$t_{closest,h} = -\frac{\langle p_{rh0}, v_{rh0}\rangle}{\|v_{rh0}\|^2} \tag{10}$$

Since, $t_{closest,h} \geq 0$, $\langle p_{rh0}, v_{rh0}\rangle \leq 0$. This implies that $(Cos\ \alpha)_0 \leq 0$. Hence, $\alpha_0 \geq \pi/2$ or $\theta_0 \leq \pi/2$. Substituting $d_{closest,h}$ and $t_{closest,h}$ in (9), we obtain -

$$d^2_{closest,h} = \|p_{rh0}\|^2 - \frac{\langle p_{rh0}, v_{rh0}\rangle^2}{\|v_{rh0}\|^2} \tag{11}$$

Since, for conflict, $d^2_{closest,h} \leq D^2$, we get -

$$1 - \frac{D^2}{\|p_{rh0}\|^2} \leq \frac{\langle p_{rh0}, v_{rh0}\rangle^2}{\|p_{rh0}\|^2\|v_{rh0}\|^2} \tag{12}$$

The right hand side is $(Cos\ \alpha)^2_0$ and since $(Cos\ \alpha)_0 \leq 0$, we obtain the $conflict$ condition as-

$$\boxed{\frac{\langle p_{rh0}, v_{rh0}\rangle}{\|p_{rh0}\|\|v_{rh0}\|} \leq -\sqrt{1 - \frac{D^2}{\|p_{rh0}\|^2}}} \tag{13}$$

2. *Time of Conflict*

Next, we find the time interval $[t_1, t_2]$ for which A_o and A_i satisfying (13) will be in conflict. Substituting the condition $p^2_{rh} \leq D^2$ in (9) and solving the quadratic equation thus obtained we get

$$t_1 = -\frac{\langle p_{rh0}, v_{rh0}\rangle}{\|v_{rh0}\|^2} - \frac{\sqrt{D^2 - \|p_{rh0}\|^2(1 - Cos^2\alpha)}}{\|v_{rh0}\|} \tag{14}$$

$$t_2 = -\frac{\langle p_{rh0}, v_{rh0}\rangle}{\|v_{rh0}\|^2} + \frac{\sqrt{D^2 - \|p_{rh0}\|^2(1 - Cos^2\alpha)}}{\|v_{rh0}\|} \tag{15}$$

Since, we look for solutions in $[0, \tau]$,

$$if\ t < t_1\ or\ t > t_2,\ \|p_{rh}\| > D \tag{16}$$

3. *Conflict Resolution*

We make the following assumptions on the system:

- Every sUAS gets a unique start time t_{start} when it enters the cruise altitude.
- An sUAS has higher priority if it has an earlier t_{start}.
- When there is a conflict the lower priority sUAS performs the resolution maneuver.
- When not undergoing a resolution maneuver, every sUAS travels at the same altitude.

The resolution in a conflict is that the sUAS with lower priority sinks to a lower altitude to prevent collision, if the time required for resolution is less than the time to imminent collision and then returns to its altitude once the collision is avoided. Figure 8 shows the three maneuvers that A_o undergoes to avoid collision, which we call *Avoidance*, *Leveling* and *Recovery* in that order.

We define additional variables as follows:

- a_{down} - Maximum vertically downward acceleration that A_o can apply
- a_{up} - Maximum vertically upward acceleration that A_o can apply
- a_{vd} - Actual vertically downward acceleration that A_o applies
- a_{vu} - Actual vertically upward acceleration that A_o applies

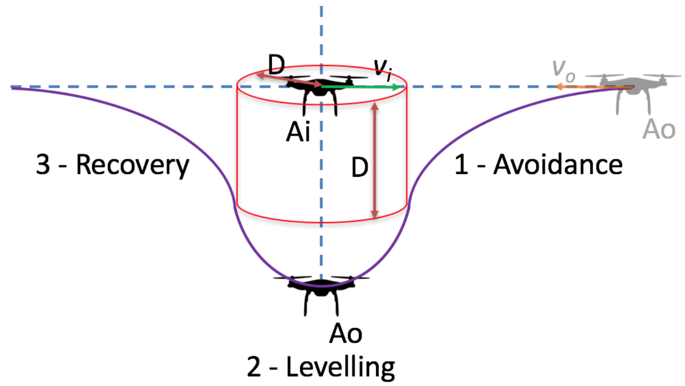

Figure 8. sUAS Tactical Resolution (Side View). A_o - Own sUAS, A_i - Intruder sUAS

For tactical collision avoidance, we assume that $a_{vd} = a_{down}$. The time durations required for the three maneuvers can now be computed.

$$(Avoidance),\ t_{Avd} = \sqrt{\frac{2D}{a_{vd}}} \tag{17}$$

$$(Levelling),\ t_{Lvl} = \frac{2\sqrt{2a_{vd}D}}{a_{vu}} \tag{18}$$

$$(Recovery),\ t_{Rec} = \sqrt{\frac{2D}{a_{vd}}} \tag{19}$$

where, $a_{vu} = \min\ (a_{up}, \frac{2\sqrt{2a_{vd}D}}{t_2 - t_1})$

A_o will initiate resolution only if,

$$t_{Avd} \leq t_1 \tag{20}$$

The time instants for different phases of the resolution are therefore as follows:

$$t_{Start} = t_1 - t_{Avd} \tag{21}$$

$$t_{EndAvd} = t_1 \tag{22}$$

$$t_{EndLvl} = t_1 + t_{Lvl} \tag{23}$$

$$t_{End} = t_{EndLvl} + t_{Rec} \tag{24}$$

where, $\{t_{Start},\ t_{EndAvd},\ t_{EndLvl},\ t_{End}\} \in [0, \tau]$

Theorem: For two sUAS A_o and A_i satisfying 13 and 20, given:

$$\begin{bmatrix} p_r \\ v_r \end{bmatrix} = \begin{bmatrix} p_{r0} + v_{r0}t + \frac{1}{2}u_r t^2 \\ v_{r0} + u_r t \end{bmatrix} \tag{25}$$

if,

$$p_{rv0} = 0 \tag{26}$$

$$u_r = [0\ 0\ u_v] \text{ for } t \in [t_{Start}, t_{End}] \tag{27}$$

where,

$$u_v = \begin{cases} -a_{down}, & \text{for } \ t \in [t_{Start}, t_{EndAvd}) \\ a_{up}, & \text{for } \ t \in [t_{EndAvd}, t_{EndLvl}) \\ -a_{down}, & \text{for } \ t \in [t_{EndLvl}, t_{End}) \end{cases} \tag{28}$$

then for $t \in [0, \tau)$,

$$p_o \notin C_i \tag{29}$$

Proof-

Recall that $p_o \in C_i$, if $\|p_{rh}\| \leq D$ & $\|p_{rv}\| \leq D$. Since, $t_{EndAvd} = t_1$ & $t_{EndLvl} \geq t_2$, from 16, for $t \in [0, t_{EndAvd})$ & $t \in [t_{EndLvl}, \tau)$, $\|p_{rh}\| > D$ i.e. $p_o \notin C_i$.

By construction, for $t \in [t_{EndAvd}, t_{EndLvl}]$, $\|p_{rv}\| > D$ i.e. $p_o \notin C_i$. Hence, for $t \in [0, \tau)$, $p_o \notin C_i$

4. Resolution Algorithm

We now present the resolution algorithm that executes the tactical collision avoidance for two sUAS described above in a simulation environment.

The *range* around an sUAS that it needs to check for conflicts with other higher priority sUAS is given by

$$range = D + (max\ v_r) \cdot t_{Avd} \tag{30}$$

Now, at a given time t in the simulation, let A = $[A_1, A_2, ... A_n]$ is the set of sUAS active at time t, p = $[p_1, p_2, ... p_n]$ is the set of updated positions of the active sUAS at time t and v = $[v_1, v_2, ... v_n]$ is the set of velocities of the active sUAS at time t.

for $i = n$ to 2, step -1 **do**

  for $j = i - 1$ to 1, step -1 **do**
    $p_r = p_i$ - p_j

    if $|p_r| \leq range$ **then**
      $v_r = v_j$ - v_i

      if $\frac{p_r \cdot v_r}{|p_r| \cdot |v_r|} \leq -\sqrt{1 - \frac{D^2}{|p_r|^2}}$ **then**
        $Evaluate\ t_1,\ t_2 and t_{Avd}$

        if $t_{Avd} < t_1$ **then**
          $Evaluate\ t_{Start}, t_{EndAvd}, t_{EndLvl}, t_{End}$
          $t_{resolve}(A_i) = t_{End} - t_{Start}$
        end if
      end if
    end if
  end for
end for

The $t_{resolve}$ above is the time for which the sUAS will undergo the entire resolution. The actual maneuvers are therefore accounted for in the position update and the $t_{resolve}$ gets updated to 0 once the resolution is over. By virtue of the construction, this algorithm ensures that every two vehicle conflict that is detected and can be resolved gets resolved assuming the higher priority vehicle continues on its path.

It is noteworthy that collisions that will occur in the simulation therefore are all forced. In other words, all resolvable two sUAS conflicts detected will be resolved but collisions will occur if one of the following conditions happens – an sUAS takes off into a conflict for which it does not have enough resolution time (see[34] for measures to prevent takeoffs into conflicts), an sUAS enters a conflict with a different sUAS before finishing its current resolution and therefore cannot avoid a collision with that and finally the higher priority sUAS finds itself in conflict with another even higher priority sUAS and initiates resolution before the lower priority sUAS has finished its own.

B. Simulation

To estimate low-altitude airspace capacity, we need to investigate how different metrics behave w.r.t. the sUAS traffic intensity. So, we require a simulator that can simulate at least until the break-point where the metric exceeds its acceptable limit.

First, we have to choose a data structure to store sUAS static and dynamic data. An ideal choice should enable fast access and update times as well as the best possible search complexity adequate to the CD&R algorithm chosen. With the proposed CD&R algorithm in the previous subsection, the repeated distance comparisons consume the most time. A simple linear search would produce a worst case search complexity of $O(n^2)$ in the number of active sUAS at every instant of the simulation time. Use of spatial databases could speed this up at the same time making it easier to incorporate geo-spatial data on static and dynamic near ground obstacles that become more crucial for sUAS with increasing fidelity of simulations.

Next, the simulator requires a trip generator that generates the origin destination pairs for sUAS trips based on the chosen underlying spatial stochastic process and the start times for sUAS trips based on the chosen temporal stochastic process. The trip generator feeds into the aircraft database whenever new trips are generated.

Finally, the simulator requires three other functions – one to define the metrics to be computed, another that runs the CD&R algorithm on the sUAS traffic and evaluates the metrics and a third one for post processing the generated data and results.

In our work, we developed a simple simulator meeting most of the above requirements. Following our approach from,[3] we consider the traffic setup as described in the introduction. Airspace is modeled as a cuboidal volume LWH defined by a rectangular area extruded to a given height H. Each sUAS is a quadruple (o, d, h, t) i.e. it has an origin, destination, height and start time. A typical flight is shown in figure 9. Each sUAS is defined as a Matlab class with properties that include the start time, origin, destination and so on. The flights' origins and destinations were generated randomly based on the population density over the rectangular area. This preserves the actual shape of the geographical area and the volume of airspace used.

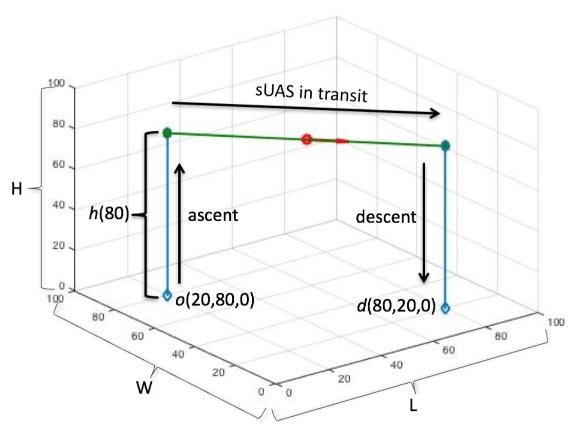

Figure 9. A typical sUAS flight path

The total number n of flights expected during the day was given, and the intensity of the traffic starting or ending at a point p of the domain was proportional to the population density at p (that is, the starting times of the flights from p form a Poisson process with the rate proportional to the density). At every instant of simulation time, handles to only the active trips were stored and accessed reducing the search space. However to implement the CD&R algorithm, the simple linear search algorithm was used producing a worst case time complexity of $O(n^2)$ in the number of steady state active trips.

We used this setup to run simulations for Bay Area (fig.3) varrying traffic densities from 100 to 220,000 flights per day and four cases of collision distance (2.5m, 5m, 10m and 20m). All sUAS travel at a uniform speed and have a maximum vertical acceleration/deceleration of 0.5g ($\sim 5ms^{-2}$). We now present the results for the different metrics.

VI. Results

The results in this section quantify the capacity estimates from the two metrics. We first present the results for the Safety metric in figure 10. We observe a threshold around 5000 flights per day for Safety being violated (flight loss per flight hour exceeding the allowable limit) with high probability. At high tolerance (very close allowable collision proximity of 2.5m), the capacity regime lies between 5000 to 20,000 flights per day. As we reduce the tolerance, meaning collisions occur at higher distances, the capacity regime comes down. For the lowest tolerance (20m), the capacity regime lies between 1000 to 5000 flights per day.

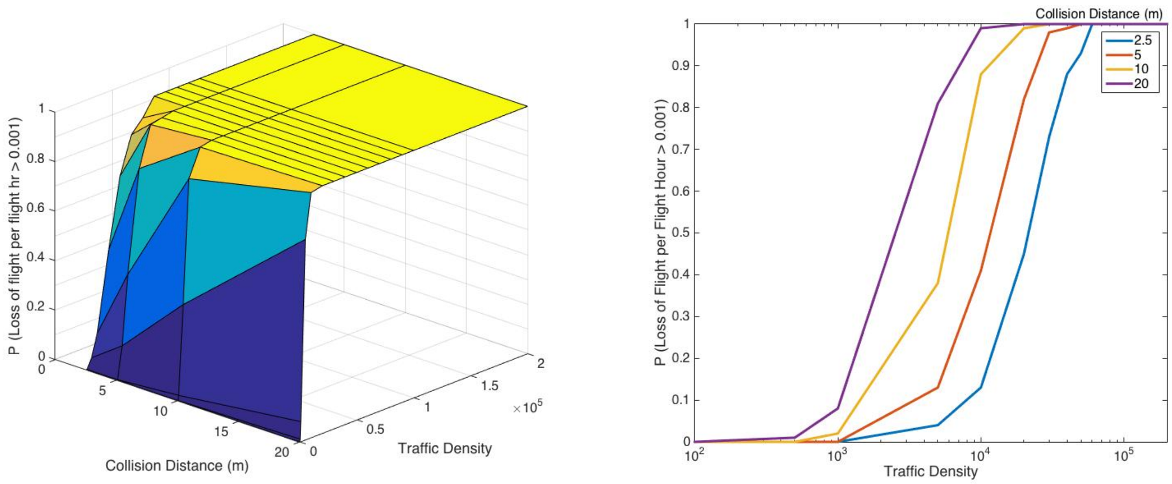

Figure 10. Probability of Loss of Flight per Flight Hour exceeding allowable limit with varying traffic density and collision distance tolerance for Bay Area

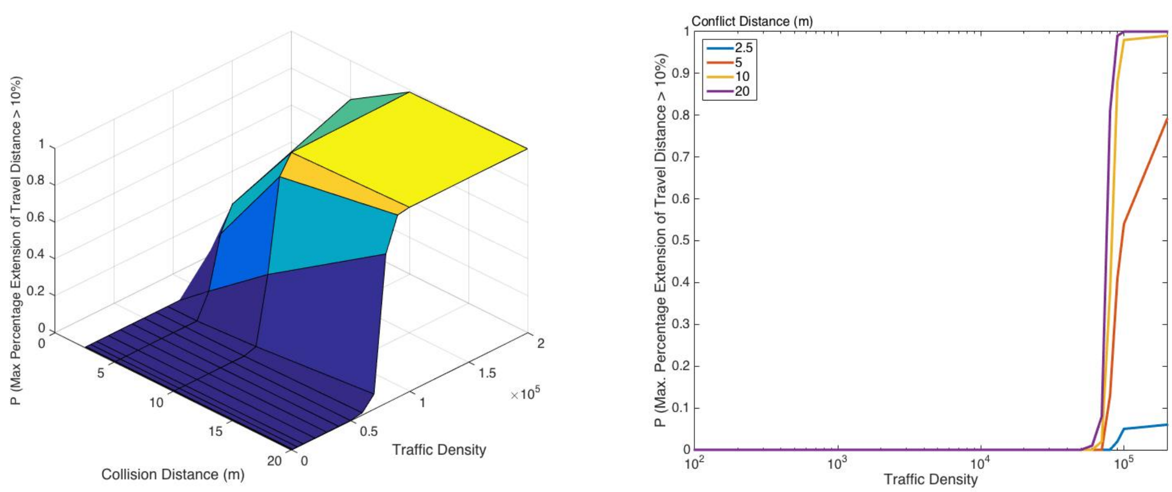

Figure 11. Probability of Max. Extension of Travel Distance exceeding allowable limit with varying traffic density and collision distance tolerance for Bay Area

The implications of this are two fold. First, it shows that some form of traffic management will become necessary beyond 5000 flights per day. Secondly, it also shows that CDRA algorithms and better trajectory following methods that reduce deviations from flight path would improve the capacity of the airspace.

Next we present the results for the Performance Metric. Here we observe that on average, the travel distance of sUAS is not extended more than 10% ever even at 200,000 flights a day. However, when we look at the maximum percentage extension observed, we again see a threshold, albeit at a much higher traffic density (over 80,000 flights a day in the worst tolerance case). 11 However, on closer observation if we look at the distribution of the percentage extensions, we see that most of the flights do not deviate much and hence the mean always stays below the allowable limit. But a few that get affected, get affected by a lot. We show this behavior by plotting a histogram of the distribution for the 100,000 flights a day case with 20m tolerance. Figure 12 shows that there are over 20 flights which travel more than double the distance. This shows that there is some unfairness in the system. Most do not get hurt but those who do get so badly.

We therefore draw two further conclusions. First, safety is a bigger bottleneck than performance with regard to airspace capacity. Secondly, there is airspace management also required to ensure a fair provision of airspace to all users.

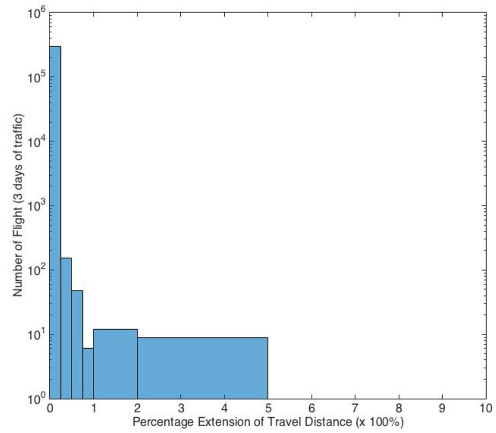

Figure 12. Distribution of Percentage Extension of Travel Distance over 3 days of traffic

VII. Conclusions

We have estimated airspace capacity for sUAS traffic in the low-altitude airspace under the consideration of Safety and Performance metrics. Results show that a capacity of 5000 sUAS flights per day can be safely achieved over a metropolitan region such as the Bay Area. The probabilities of these metrics exceeding their allowable limits show sharp thresholds at their respective capacities. Modes of operations with traffic densities and tolerances in the blue areas on Figure 10 and Figure 11 are very unlikely to exceed the capacity, while operating in the yellow areas will almost surely exceed the airspace capacity. In general graphs like these for a variety of metrics will help the authorities in quantifying the tradeoffs between the allowable density of the sUAS traffic and the conflict detection and resolution capabilities (collision tolerance). Next, we find that between safety and performance, Safety is a bigger limiting factor for sUAS. Secondly, although average performance may not be bad at high traffic densities but the distributions show that the system becomes highly unfair with those losing performance being severely impacted at high traffic densities. Both of these suggest that some form of traffic management becomes essential in near future if the future unmanned flights need to be enabled in the existing airspace.

Finally, it is of interest to estimate the airspace capacity for other values of allowable limits M', and more generally – under a wider range of metrics M. One of the most important concerns that the UTM community is currently facing is to measure the volume of unmanned aircraft that can be accommodated in the existing airspace based on considerations of safety, performance, spectrum required for communication and noise levels. Our definition has enabled us to provide a stepping stone to respond to that concern at least on grounds of safety, performance and noise.[2] Airspace capacity estimates based on the other afore-mentioned considerations will improve this further.

Acknowledgment

We express our sincere gratitude to Dr. Alex A. Kurzhanskiy from UC, Berkeley, Dr. Parimal H. Kopardekar from the NASA Ames Research Center, and Professor Jacco Hoekstra from TU Delft for valuable insights on the basic ideas. We thank Zhilong Liu, fellow PhD student at UC Berkeley for contributions to earlier approaches to the simulator used in the current work. We also acknowledge Mr. Frank Ketcham from Delta Airlines for teaching us about different air traffic management systems. The work of LS and VP is part of UTMOK and UTM50 projects supported by the Swedish Transport Administration (Trafikverket) via the Swedish Air Navigation Service provider LFV (Luftfartsverket).

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